CS125 Final Lecture

Areas of TCS

(Theoretical Computer Science)

Areas of TCS

- Algorithms
- Data structures
- Complexity Theory
- Computer Science + Economics
- Cryptography and Privacy
- Computational Learning Theory
- Coding Theory
- Quantum Computing

Areas of TCS

- **Algorithms**
  - Word RAM
  - Graph algorithms
  - Algorithmic spectral graph theory
  - Algorithmic linear algebra
  - Distributed algorithms
  - Parallel algorithms
  - Property testing (like the BLR linearity test)
  - Streaming algorithms
  - Online algorithms
  - Approximation algorithms
  - External-memory / cache-oblivious
  - Computational geometry
  - Number-theoretic problems (e.g. primality testing)

Areas of TCS

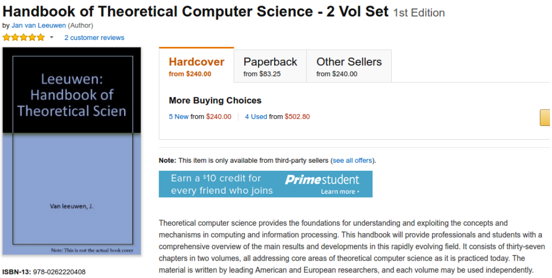
- **Data structures**
  - Many qualifiers: amortized/worst case, static/dynamic, persistent/ephemeral, randomized/deterministic
  - Tradeoffs (time vs. space, or update vs. query time), upper and lower bounds
- **Complexity Theory**
  - Concrete complexity (communication complexity, branching programs, circuits, formulae, ...)
  - Pseudorandomness
  - Algebraic complexity
  - Proof complexity
  - Interactive proof systems
- **Computer Science + Economics**
  - Algorithmic mechanism design
  - Algorithmic game theory
- **Cryptography and Privacy**

Areas of TCS

- **Computational Learning Theory**
- **Coding Theory**
- **Quantum Computing**
  - Quantum error-correction
  - Quantum communication complexity
  - Quantum complexity ("quantum Turing machines")
  - Quantum algorithms

Also “Theory B”

Theory B



Volume A *covers models of computation, complexity theory, data structures, and efficient computation in many recognized subdisciplines of theoretical computer science.* **Volume B** *presents a choice of material on the theory of automata and rewriting systems, the foundations of modem programming languages, logics for program specification and verification, and several chapters on the theoretic modeling of advanced information processing ...*

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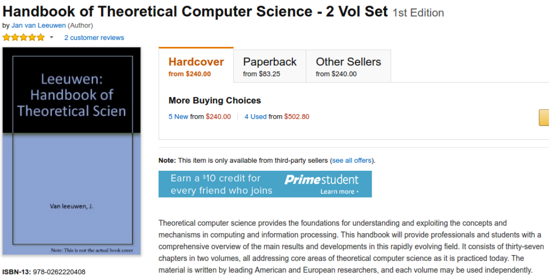

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Take for example: CS152 (Programming Languages), CS252r

Back to Theory A

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Word RAM

- Seen in this course: hashing, counting sort, etc.
- Mentioned throughout semester:
    - Dynamic predecessor in $O(\lg w)$ [van Emde Boas'75]
    - Least significant bit in $O(1)$ time [Fredman, Willard'90]
    - sorting in $O(n\sqrt{\lg \lg n})$ time randomized [Han, Thorup'02]
    - sorting in $O(n \lg \lg n)$ time deterministic [Han'02]
    - Min spanning tree in $O(m + n)$ deterministic [Fredman, Willard'94]
    - Undirected single source shortest paths in $O(m + n)$ [Thorup'99]
    - Directed SSSP in $O(m + n \lg \lg n)$ [Thorup'04]
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Can see some more instances of "the power of word RAM" in CS224 and 6.851 (MIT).

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- **Algorithmic spectral graph theory**
  - Can say a lot about graphs by looking at eigenvectors and eigenvalues of matrices.
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  - **Recently:** "higher-order Cheeger". λ_k small iff G can be partitioned into k clusters without many edges crossing the clusters. [Lee, OveisGharan, Trevisan'12], [Kwok, Lau, Lee, OveisGharan, Trevisan'13], . . .

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  - $L = \sum_{e \in E} b_e b_e^T$ (for $e = (u, v)$, $b_e = \mathbf{1}_u - \mathbf{1}_v$ for $u < v$)
  - For $x \in \{-1, 1\}^n$, $\frac{1}{4} x^T L x$ is size of cut
  - **Thm:** Can sample only $O(n \lg n)$ edges s.t. new $\tilde{L}$ satisfies $x^T \tilde{L} x \approx x^T L x$ for all $x \in \{-1, 1\}^n$ [Benczúr, Karger'96]

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  - Ideas eventually led to solution of 57-year old "Kadison-Singer" problem in functional analysis [Marcus, Spielman, Srivastava'15]

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  - **Low-rank approximation:** Given $A \in \mathbb{R}^{n \times d}, k \geq 1$, compute $A_k = argmin_{rank(B) \leq k} \|A - B\|$
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Streaming algorithms

A (fake) search engine query log from Nov 7th:

18:58:02	`wikileaks`
18:59:12	`mlb playoffs`
19:07:40	`gmail login`
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Finding frequent items

Problem: Given stream of items (e.g. words) coming from some universe $\mathcal{U}$ (e.g. English dictionary), report a small list $L \subset \mathcal{U}$ containing all "frequent" items

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- **Goal:** using $\ll n$ memory, output small such L (e.g. $|L| \leq \frac{10}{\varepsilon}$)

Harder problem: change detection

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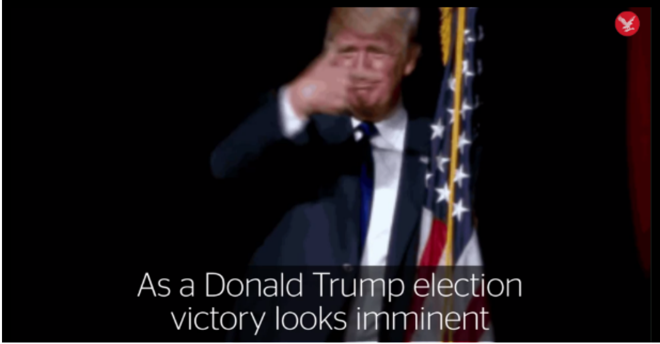
News › World › Americas

How to move to Canada: Immigration website crashes as Donald Trump becomes 45th US President

Moving to live in Canada is not easy but not impossible either

Andrew Griffin | @_andrew_griffin | Tuesday 8 November 2016 | 395 comments

46K shares



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Goal: use low memory to find small list of items that had large frequency changes.

In general

Streaming algorithms: make one pass over a massive dataset while answering queries, using memory *sublinear* in the data size.

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Take CS229r (Algorithms for Big Data), CS222 (Algorithms at the End of the Wire), or "Sublinear Algorithms" at MIT.

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See "Communication complexity" book by Kushilevitz and Nisan. Also CS229r (Information Theory in Computer Science).

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Take "Proofs, beliefs and algorithms through the lens of Sum of Squares" at Harvard/MIT.

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  - **PPAD:** The class for which the following problem is complete: given digraph G implicitly s.t. each vertex has at most one outgoing and at most one incoming edge. Given $s \in V(G)$ and a *description* of a poly-time computable function $f(v)$ to compute successors/predecessors, find a sink. [Papadimitriou'94]

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  - Businesses: Maximize revenue?
  - Government: Maximize "social welfare" (total happiness)?

Take CS134 (Networks), CS284r (Social Data Mining), CS284r (Incentives and Information in Networks). See more at `econcs.seas.harvard.edu`. Also 6.891 (Topics in AGT) and 6.853 (Games, Decision, and Computation) at MIT.

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  - **Private-key**: Alice+Bob share secret, communicate covertly.
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  - **Security multiparty computation**, **Functional encryption**, **Private information retrieval** ...

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Take CS127 (Intro to Crypto), CS227r (Topics in Crypto+Privacy), CS229r (Mathematical Approaches to Data Privacy). Also 6.875 (Crypto) and 6.876 (Topics in Crypto) at MIT.

Areas of TCS

- **Computational Learning Theory**
- **Coding Theory**
- **Quantum Computing**
  - Quantum error-correction
  - Quantum communication complexity
  - Quantum complexity ("quantum Turing machines")
  - Quantum algorithms

Computational Learning Theory

- **PAC learning.**
- Distribution $\mathcal{D}$ over $\mathcal{X} \times \mathcal{Y}$, and "concept class" $\mathcal{F}$ which is subset of functions from $\mathcal{X}$ to $\mathcal{Y}$.
- Given many iid samples from $\mathcal{D}$, "learn" the best $f \in \mathcal{F}$ minimizing $\mathbb{P}_{(x,y)\sim\mathcal{D}}(f(x) \neq y)$ [Valiant'84]

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- **Statisical query model.** Weaker than general PAC learning: learner only allowed to make statistical queries to $\mathcal{D}$ (i.e. can ask oracle some query $\phi : \mathcal{X} \to [-1, 1]$ and get back an estimate of $\mathbb{E}_{x\sim\mathcal{D}}\, \phi(x))$ [Kearns'98].

CS228 (Computational Learning Theory). Also, book by Kearns and Vazirani.

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- Cryptography
  - **Quantum money.** Bank notes that are guaranteed to be unforgeable assuming laws of quantum mechanics [Wiesner'83], several later papers by Scott Aaronson and collaborators.
  - **Quantum key distribution.** Alice+Bob share secret keys via public communication; eavesdropping adversary Eve learns nothing [Bennet, Brassard'84], [Ekert'91].

Quantum Computing

- Complexity
  - **Quantum circuits.** Take input that is n "qubits", not bits. n qubits just means a vector $x \in \mathbb{C}^{2^n}$ with $\sum_i |x_i|^2 = 1$ (so probability distribution over $\{0,1\}^n$). Allowed to do unitary operations on x, i.e. $x \mapsto Ux$ for $U^*U = UU^* = I$. Can "measure" and collapse to a basis state given this probability distribution.
  - Basis for classical circuits: AND, OR, NOT. For quantum: gates apply unitary matrix to input; has been shown a finite set of gate types suffices to do quantum computation. (universal set of gates S satisfies $\forall n \geq n_0,$ subgroup generated by S is dense in group of unitary matrices with determinant 1 operating on n qubits)

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  - **BQP**. Languages decided by uniform poly-size quantum circuits. $\mathbf{P} \subseteq \mathbf{BPP} \subseteq \mathbf{BQP} \subseteq \mathbf{PP} \subseteq \mathbf{PSPACE}$.
  - Some problems known to be in **BQP** but unknown if in **BPP**, e.g. FACTORING [Shor'94].

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  - **Quantum error-correction.** Need to store data being computed on robustly due to quantum decoherence and faulty gates. Some physical barriers though ("no-cloning theorem" [Wootters, Zurek'82], [Dieks'82])
  - ...

The End

See more TCS courses at:

- http://toc.seas.harvard.edu/toc-courses
- http://econcs.seas.harvard.edu/teaching
- http://toc.csail.mit.edu/

The End

See more TCS courses at:

- `http://toc.seas.harvard.edu/toc-courses`
- `http://econcs.seas.harvard.edu/teaching`
- `http://toc.csail.mit.edu/`
- Also: AM 106/206 (Applied Algebra), AM 107 (Graph Theory & Combinatorics), MATH 155r (Combinatorics), ES 250 (Information Theory), MATH 141 (Intro to Mathematical Logic), Philosophy 144 (Logic & Philosophy)
- Connections to other areas: natural language processing, circuit design, parsing and compiling, programming languages, artificial intelligence ...

And check out our seminars / explore research opportunities!

`http://toc.seas.harvard.edu/events-seminars`