Robot Search in 3D Swarm Construction

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Abstract

Robots in a swarm construction system need to be able to find their way to all potential places to add material to a structure. In two dimensions, a (trivial) procedure exists for a stateless robot moving along the outside of a structure, that will let it start at any point, take it to all potential attachment sites exactly once, and return it to its starting point, regardless of what part of the structure has already been built. Here we show that for three-dimensional structures made from cubic building blocks, where each exposed face of a block is a potential attachment site, no such procedure exists. Any robot search scheme must then necessarily involve excess movement and/or increased component complexity (e.g., memory or communication).

1. Introduction

A swarm construction system is one in which a large, unspecified number of autonomous robots collectively build a structure according to some set of user specifications. This framework belongs to the class of self-organizing systems, engineered in such a way that the independent actions of many simple interacting components produce a desired high-level result. In this paper we prove a geometric result about structures made from cubic building units, which implies for three-dimensional systems that efficiency and simplicity are not simultaneously attainable.

Swarm systems might one day be used for construction in settings difficult or dangerous for humans, e.g., building habitats in extraterrestrial or underwater environments to await human travelers, or shoring up partially collapsed structures in disaster areas. Other applications might include highly flexible fabrication of smaller-scale artifacts, as in a complementary approach to current rapid prototyping methods. The swarm approach can potentially provide many advantages over centralized systems, including parallelism, scalability, and robustness.

The robots' task in such a system can be broken into two independent parts:

1. Find all sites where material might potentially be added (the *search problem*).

2. Determine, for any such site, whether adding material there is consistent with eventually achieving the structure desired. This issue comprises both whether material should ultimately appear at a site, and whether adding it there now, given the current state of the partial structure, will later prevent robots from reaching other sites where material is desired.

In previous work [6–9] we have focused on the second part of the problem, which depends heavily on such features of the system as movement constraints of robots and building material. Here we consider only the first part, the search problem of finding sites for potential attachment.

We discuss the case where the building material consists of square or cubic blocks, and robots are able to maneuver along the surface of the structure [1–9]. To avoid unnecessary movement (and associated waste of time and energy), it is desirable for robots to minimize the number of times they repeatedly visit sites that they previously visited and rejected. The ideal path would take a robot to visit each potential attachment site exactly once, starting from any point on the structure surface. Such a sequence would be a Hamiltonian path on a graph where nodes correspond to potential attachment sites, and edges connect physically adjacent sites. If the path is static and robots may start anywhere, the path should be a cycle to ensure that all sites are visited.

For two-dimensional structures, there exists a trivial procedure robots can follow to take such a path: following the perimeter of the structure in a given direction, say clockwise, will take a robot to each site once and return it to its starting point (Figure 1). Whether such a procedure exists for three-dimensional structures is less clear. The "hairy ball theorem", which states that there is no vector field tangent to the surface of a sphere which is nonzero everywhere, suggests that no such cycle exists in a domain where sites are continuous: a robot following any path will eventually

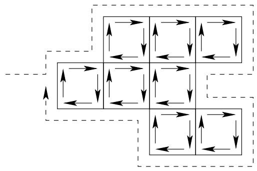

Figure 1. No matter where a robot starts along the perimeter of a 2-D structure, it will visit each potential attachment site exactly once and return to its starting point (dotted line) if it moves clockwise at every step (or, equivalently, follows clockwise-pointing arrows drawn on every block).

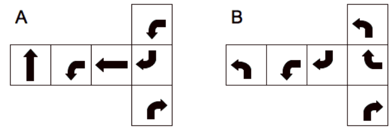

Figure 2. The two possible Hamiltonian cycles on the surface of a cube (unfolded). A robot following such a cycle will repeatedly go through the sequence of moves (A) {left, right, straight} or (B) {left, right}. (A third possibility is a reflection of the first: {right, left, straight}.)

reach a point where it must stop (or else return to a previously visited location) before returning to its origin. However, when sites are discrete, the theorem does not apply. For instance, multiple Hamiltonian cycles exist to visit all the faces of a cube (Figure 2).

Even if such a path exists, the problem remains of a robot finding it. Some approaches might require very significant capabilities from the robot in terms of perception, memory, and/or computation, and may not scale well with structure size. Other approaches may make use of complex building material, e.g., blocks with embedded processors and the ability to communicate [3, 9]. For greatest simplicity and lowest cost, it would be best if a stateless robot could determine how to follow the path at any point based only on very limited local information.

Information left in the environment—the "stigmergy" often used in swarm systems and inspired by social insects—can be of great use to simple robots. In particular, a path could in effect be drawn on the surface of the structure, telling a robot crawling along the surface which way to go next. If cues of this sort are static, then adding any new block to the structure needs to result in an appropriate path without requiring an update to any of the rest of the existing path.

The question of interest, then, is whether a fixed scheme exists for coding and assembling a set of blocks such that (1) a Hamiltonian cycle exists for any structure at any stage of completion, and (2) attaching another block at any site still gives a Hamiltonian cycle.

For two-dimensional structures of square blocks, such a scheme is to draw a clockwise-pointing arrow on each face of each block (Figure 1). A robot whose movement is determined by following these arrows will take a Hamiltonian path around the perimeter of the structure no matter where it starts. Attaching a block to the structure at any site in any orientation (assuming blocks cannot be rotated through the third dimension, making the arrows point counterclockwise) will yield a structure with a Hamiltonian cycle.

In the next section, we will show that no such scheme exists for three-dimensional structures of identical cubic blocks. As a result, any sufficient approach to the search problem necessarily must involve repeated visits to potential attachment sites (as with a random walk), potentially complex robots (as with a systematic search that maintains a history of visited sites), and/or communication between blocks (as with following a dynamic gradient maintained by the blocks of the structure). Previous work [9] explores three such approaches and the tradeoffs between them.

2. Three-dimensional analysis

A structure is built starting from a single block. Thus whatever path is drawn on the surfaces of all blocks must constitute a Hamiltonian cycle for each block in isolation. It is easy to show by exhaustion that there are only two such cycles on a cube, shown in Figure 2.

When connecting a block to a face of an existing structure, the new block needs to be oriented in such a way that the arrows on the block are consistent with the arrows on the structure. The head of every arrow on the structure that points the way onto the block should point to the tail of an arrow on the block, and vice versa (Figure 3), in order for the paths represented by arrows to be continuous everywhere. Otherwise, a path will be broken; two arrows will point to the same face, and some face will have no arrows pointing to it, inconsistent with a Hamiltonian cycle.

The unique way to ensure matching up two blocks and their arrows in this way is to have identical arrows on the faces that are brought into contact, and orient them head-to-tail. With this rule, the arrow that led into the joined face on one block will lead into the arrow that led out of the joined face on the other block.

In section 2.1 we show that for path A in Figure 2, this rule cannot in general be followed to allow a new block to be attached to a structure consistent with all arrows already

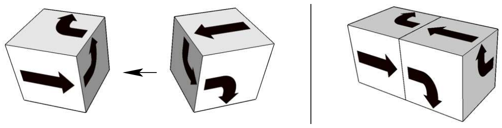

Figure 3. An example of attaching two blocks such that their arrows are consistent. The faces brought into contact each have left arrows, in opposite orientations. The resulting structure has a single Hamiltonian cycle drawn on its surface.

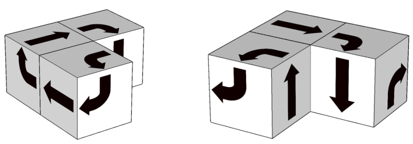

Figure 4. Two views of an example structure built of cubes encoded with path A, where there is no way to attach a fourth cube at the corner site consistent with both of its neighbors.

present. In section 2.2 we show that for path B, a new block can always be added in this way, but the result is not always a single Hamiltonian cycle. Therefore there is no way to encode static cubes such that they can be used to build a structure where, at every stage of completion, a stateless robot moving along the surface can always follow a path visiting every exposed face exactly once.

2.1. Path A

Figure 4 shows an example demonstrating that path A of Figure 2 can lead to situations where no cube can be attached at a site so as to be consistent with all its neighbors. Three blocks have been connected according to the attachment rule and consistent with their neighbors, and the arrows on the surface form a Hamiltonian cycle. However, no block can be attached at the empty site with two neighbors: the two neighboring faces both have straight arrows, inconsistent with path A. (In fact, there is no way to draw arrows on a cube such that, if it were attached at that site, the four-block structure would have a Hamiltonian cycle on its surface.) Path A, then, cannot in general be used to build a structure such that a Hamiltonian cycle is present at every step of construction.

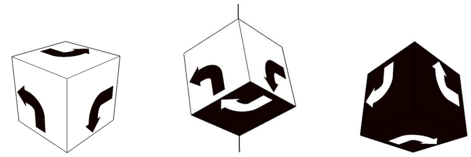

Figure 5. Three views of a cube patterned with path B. The cube has threefold rotational symmetry about the axis shown (middle).

2.2. Path B

Coloring left-arrow faces white and right-arrow faces black, as in Figures 5, 7, and 8, makes several properties of this path clearer. The three white faces border one vertex of the cube, with arrows directed outward; the three black faces border the diagonally opposite vertex, again with arrows directed outward (Figure 5). The block has threefold rotational symmetry about the axis that passes through these two vertices. Thus a block in a cubic lattice has exactly eight possible distinct orientations; the “white” vertex can be in any of the eight corners.

Because of the attachment rule about matching up arrows, the orientation of one block determines the orientation of its neighbors, which determine the orientation of their neighbors, and so on. It is easy to show that all of these orientations are mutually consistent: it will always be possible to attach a block to a structure so that there are no broken paths, and the head of every arrow on every face will point to the tail of an arrow on a neighboring face.

All of space can then be consistently tiled with blocks marked with path B; the orientation of each is uniquely determined by that of the first. Figure 6 shows how these orientations fit together. An interesting property is that no matter how the structure is built, any exposed faces in the same plane will always be the same color (Figure 7).

While it is always possible, using path B, to attach blocks anywhere so as to be consistent with the existing structure, the resulting paths are not always Hamiltonian. Adding a block may end up splitting an existing cycle into separate cycles (Fig. 8). A robot following such a path would return to its starting point before visiting all potential attachment sites. As a result, path B is also unsuitable for building a structure so as to provide a Hamiltonian cycle at every stage.

3. Conclusion

We have shown that no way of drawing a static path on identical cubic blocks will let them be assembled into a structure such that a Hamiltonian cycle, visiting all exposed faces exactly once, will necessarily exist at every step. Thus

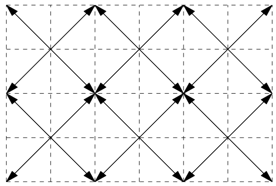

Figure 6. Tiling pattern of consistent block orientations, projected into any of the three orthogonal planes. Dashed lines show block boundaries; arrows show orientation, pointing from "black" vertex to "white" vertex (Figure 5). In three dimensions, arrows have a component pointing into or out of the plane, so that eight "black" vertices or eight "white" vertices come together at a single point.

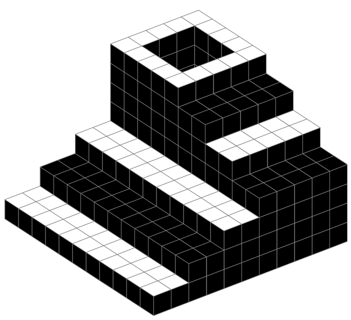

Figure 7. An example structure showing that, using path B, exposed faces in the same plane will all be the same color. (Arrows are omitted.)

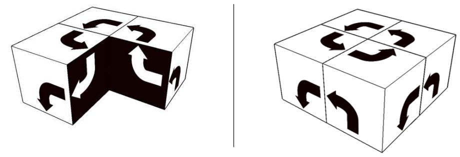

Figure 8. (Left) A three-block structure built of cubes encoded with path B, with a Hamiltonian cycle. (Right) Adding a fourth block gives a structure with two continuous but separate closed paths, one visible on top, the other winding around the sides and bottom.

it is in general impossible for stateless robots, acting on the basis of limited local information, to reliably find all potential attachment sites on a partially completed structure they build of passive cubic blocks, without revisiting sites.

A swarm construction system using cubic blocks thus requires greater complexity from its components, greater expenditure of time and energy, or both. Previous work [9] discussed three possible search schemes and explored the tradeoffs between them. One scheme is a random walk: robots simply move at each step to a random neighboring face. This approach in general results in a very great deal of unnecessary movement, more so for structures with greater surface area. A second possibility is a systematic search, where robots keep track of visited sites and choose their movement in a principled way to visit unexplored areas. While this approach can eliminate considerable revisiting of sites, some repeated movement remains; it also has robot memory requirements that increase with the size of the desired structure, and depending on the details of the search, may restrict the class of structures that can be built. A third scheme is to embed processors in the building blocks, let them communicate with physically attached neighbors, and have them direct robots straight to the nearest available attachment site. This approach requires substantially more complex blocks, and can call for a very great deal of communication among them, with associated energy costs.

It would be ideal if all these complexities could be avoided and stateless robots could find their way to all potential attachment sites without repeated movement or communication. However, some such increased complexity is necessary for any such 3D swarm construction system—regardless of the nature of the rules dictating where material may legitimately be attached.

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