EPS 237 Lesson 1: Introduction

R. Wordsworth

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1 A Toy Climate Model

A planet around a star with luminosity L_S at an orbital distance d receives a flux of visible radiation

$$F_0 = \frac{L_S}{4\pi d^2}. \tag{1}$$

Assuming thermal emission from an isothermal surface, energy balance then gives

$$\pi r_p^2 F_0 (1 - A) = 4\pi r_p^2 F_I = 4\pi r_p^2 \sigma T^4. \tag{2}$$

We write $T = T_{eq}$, defining *equilibrium temperature*

$$T_{eq} = \left(\frac{F_0(1-A)}{4\sigma} \right)^{0.25}. \tag{3}$$

For Earth with $F_0 = 1366$ W/m^2 and $A = 0.3$, we get a rather chilly $T_{eq} = 255$ K. This indicates that (3) is not a good description of Earth's surface temperature. In particular, it suggests that the atmospheric greenhouse effect is a significant factor that needs to be included.

Now let's add an atmosphere. To keep it simple, we'll assume an isothermal atmosphere at temperature T_a that is completely transparent in the visible but partially opaque in the infrared. We will neglect all atmospheric scattering for now. If the atmosphere has total infrared (IR) emissivity $\mathcal{E}_{IR}$ and total infrared absorptivity α_{IR}, its energy balance becomes

$$\mathcal{E}_{IR}\sigma T_a^4 + \mathcal{E}_{IR}\sigma T_a^4 = \alpha_{IR}\sigma T_s^4. \tag{4}$$

Asserting that $\mathcal{E}_{IR} \equiv \alpha_{IR}$[1], we get

$$\sigma T_a^4 = \frac{1}{2}\sigma T_s^4 \tag{5}$$

and hence

$$T_a = 2^{-1/4} T_s \approx 0.84 T_s. \tag{6}$$

This is a relation that turns out to be important even when we move to more realistic atmospheres. Later, we will get to know this temperature as the *skin temperature*.

In our toy model, the surface energy balance is now

$$F_0(1 - A_s)/4 + \mathcal{E}_{IR}\sigma T_a^4 = \sigma T_s^4. \tag{7}$$

[1] This comes from Kirchoff's Law, which we will cover soon. For now, we just take it as a given.

where the extra term accounts for the warming of the surface by the atmosphere. Solving, we get

$$F_0(1 - A_s)/4 = (1 - \frac{1}{2}\mathcal{E}_{IR})\sigma T_s^4 \tag{8}$$

and hence

$$T_s = \left(\frac{F_0(1 - A_s)}{2\sigma(2 - \mathcal{E}_{IR})}\right)^{1/4}. \tag{9}$$

$\mathcal{E}_{IR}$ can vary between 0 and 1. For $\mathcal{E}_{IR} \to 0$, it is as if the atmosphere does not exist, and $T_s = T_{eq}$. For $\mathcal{E}_{IR} = 1$, T_s is a toasty 303 K. What value of $\mathcal{E}_{IR}$ would be required to reproduce Earth's real mean surface temperature (288 K)?