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Spin-Wave Instabilities in Magnetic Thin Films*

J. B. COMLY, T. PENNEY, AND R. V. JONES
Division of Engineering and Applied Physics, Harvard University, Cambridge, Massachusetts

High-power resonance effects have been observed in thin films of Permalloy. These effects are characterized by a saturation of the main resonance and in some cases the growth of a subsidiary absorption peak at magnetic fields below resonance. The h_{crit} associated with the saturation of the main resonance is found to be sensitive to the film thickness. The size dependence of nonlinear effects in films seems to be associated with a distortion in the spin-mode dispersion relation at low k numbers.

THE striking consequences of the unstable growth of certain spin-wave modes have been observed in Permalloy thin films.[1] These instabilities are manifested by changes in the loss component of the perpendicular rf susceptibility χ'' at high microwave power levels. In the experiments presented here the biasing dc magnetic field was oriented within the plane of the film. A pulsed X-band magnetron was used as a source for the perpendicular rf magnetic field.

The high-power behavior of χ'' for thin Permalloy films is characterized by two distinct effects—a saturation of the main ferromagnetic resonance and in some cases the growth of a subsidiary absorption peak at smaller values of the biasing field. Figure 1 shows the typical course of the high-power effects for several films studied. The main resonance (at approximately 850 Oe) exhibits at first a rather gradual saturation and then declines steeply at very high powers. The loss at the position of the subsidiary absorption rises with increasing power and then shows some saturation at extremely high powers.

The high-power behavior of bulk ferromagnetic insulators has been beautifully interpreted within the framework of the spin wave instability theory first discussed by Suhl.[2] In the Suhl theory one must distinguish between two possible modes of the spin-wave

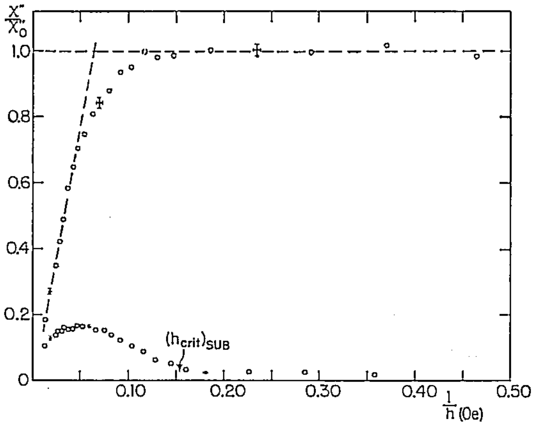

FIG. 1. The loss component of the rf susceptibility of an 85:15 Permalloy, 1200-Å film plotted against $(h_{\mathrm{rf}})^{-1}$. The upper curve represents a saturation of the 924-Oe main resonance and the lower curve represents the growth of a subsidiary absorption at a biasing field of 330 Oe.

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[2] H. Suhl, J. Phys. Chem. Solids 1, 209 (1957).

TABLE I. *X*-band resonance and nonlinearity data for various Permalloy films.

Melt composition (nickel:iron)	86:14	83:17	83:17	83:17	85:15	85:15
Approximate film thickness (Å)	4800	1200	1200	2400	1200	1200
Linewidth (Oe)	150	54	55	87	127	174
Field for main resonance (Oe)	951	821	813	810	924	846
h_{crit} for resonance (Oe)	0.7	5	4	7	16	21
$h_{\text{crit}}^{(1)}$ calculated first order (Oe)	3.6	0.5	0.5	1.2	2.5	4.7
$h_{\text{crit}}^{(2)}$ calculated second order (Oe)	18	4	4	8	14	23
Field for subsidiary absorption (Oe)	none	212	212	400	330	330
h_{crit} for subsidiary absorption (Oe)	none	6	5	3	7	17

instability—the so-called first- and second-order processes. By the first-order process a pair of spin waves degenerate with one half the microwave frequency ω grows exponentially when the strength of the driving term exceeds the characteristic losses of this particular pair. The second-order process involves the excitation of spin waves degenerate with ω itself. If $\omega/2$ is degenerate with the spin-wave manifold at ferromagnetic resonance, then the Suhl theory predicts only a saturation and broadening of the main resonance at high powers. If this condition does not hold, i.e., if the ratio $\omega/4\pi M\gamma$ is large—then the first-order process gives rise to a subsidiary absorption away from resonance and the saturation of the main resonance occurs eventually by means of the second-order process.

A direct application of the Suhl theory to the Permalloy experiments under consideration $(\omega/4\pi M\gamma\approx\frac{1}{3})$ leads one to expect a first-order saturation of the main resonance and no subsidiary absorption. The critical rf field (linearly polarized) for the onset of saturation should then be given by

$$h_{\text{crit}}^{(1)}=(1.6)(\Delta H_k)(\Delta H_0)/4\pi M. \qquad (1)$$

The second-order process would have a critical field

$$h_{\text{crit}}^{(2)}=(\Delta H_0)(\Delta H_k/4\pi M)^{\frac{1}{2}}. \qquad (2)$$

Table I presents a comparison of experimental and calculated h_{crit} values for several Permalloy samples. $h_{\text{crit}}^{(1)}$ and $h_{\text{crit}}^{(2)}$ have been calculated assuming $\Delta H_k=\Delta H_0$. In the 4800-Å film the expected first-order saturation process is indicated by a main resonance saturation with no subsidiary absorption and by a measured h_{crit} in agreement with the calculated first-order value. However, for the 1200- and 2400-Å films the measured h_{crit}'s generally agree with calculated second-order values, and we observe a subsidiary absorption as well as main resonance saturation. One must conclude from these results that the nature of the instability depends on sample thickness.

In these experiments the unstable spin modes will have wavelengths of the order of the film thickness or larger. Under these conditions the appropriate spin modes are more like magnetostatic Walker modes than the plane spin waves of an infinite medium. Damon and Eshbach[3] have shown that the limits on the magnetostatic mode spectrum in a ferromagnetic plate of thickness L, collapse at $k=0$ to the uniform precession frequency (at the top of the extrapolated spin-wave manifold). The bottom of the magnetostatic mode spectrum drops down to the bottom of the simple spin-wave manifold only for k's larger than $2\pi/L$. A detailed calculation for thin films by Harte[4] including exchange and demagnetizing effects shows a similar raising of the bottom of the mode spectrum for k's of the order of the inverse sample thickness. For Permalloy at X band, $\omega/2$ is degenerate with the bottom of the spin-wave manifold at $k=5\times10^5$ cm^{-1}. Since $k=2\pi/L$ is also 5×10^5 cm^{-1} for $L=1200$ Å, one would expect that the bottom of the mode spectrum should be above $\omega/2$, and that there should be no modes degenerate with $\omega/2$. Thus the first-order process would be disallowed at resonance. For sufficiently small biasing fields the manifold becomes low enough to allow modes to be degenerate with $\omega/2$ and a subsidiary absorption should occur. For the 4800-Å film, $k=2\pi/L$ should be small enough to allow the bottom of the mode spectrum to drop below $\omega/2$ even at resonance, allowing a first-order saturation process and no separate subsidiary absorption.

It appears that the magnetostatic distortion in the spin-wave spectrum at small k values may account for the observed thickness dependence of nonlinear microwave effects.

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[3] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).

[4] K. J. Harte, Bull. Am. Phys. Soc. 7, 448 (1962).