

# MAGNETIC ALIGNMENT OF INTERSTELLAR GRAINS

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## ABSTRACT

The Davis-Greenstein theory for the orientation of interstellar grains is reconsidered. First the physical process of orientation is discussed from a different viewpoint, based on equilibrium considerations rather than on particle dynamics. (1) The Fokker-Planck equation is used to show that for spherical grains the distribution of rotational kinetic energy about an axis perpendicular to the magnetic field,  $\mathbf{B}$ , corresponds to thermal equilibrium at a temperature between the gas kinetic temperature,  $T_g$ , and the internal grain temperature,  $T_i$ . The ratio,  $\delta$ , of the diffusion coefficients due to magnetic dissipation and to atom-grain collisions determines the precise equilibrium temperature. The distribution of rotational energy about an axis parallel to  $\mathbf{B}$ , for which magnetic dissipation vanishes, corresponds to thermal equilibrium at the gas temperature,  $T_g$ . If  $T_g$  exceeds  $T_i$  and  $\delta$  is large, the rotational momentum,  $J$ , will tend to be oriented parallel to  $\mathbf{B}$ . (2) Similar equilibrium arguments in the absence of a magnetic field show that the axis of symmetry of a rotating prolate spheroid tends to be perpendicular to  $J$ . These two results may be combined to give an approximate determination of the orientation, which agrees roughly, for weak orientation, with the value obtained from the Davis-Greenstein dynamical analysis. While the exact orientation has not been determined, the present point of view takes inverse reactions correctly into account, giving no orientation when  $T_i$  equals  $T_g$ , and seems to provide a simple, more realistic physical picture.

Second, the magnetic dissipation to be expected in different possible substances that might be present in interstellar grains is re-evaluated, using recent theories of the complex physical processes involved. For paramagnetic relaxation the imaginary part,  $\chi''$ , of the magnetic susceptibility does not differ greatly from that adopted by Davis and Greenstein. For ferromagnetic single domains, however, a detailed analysis of rotational mode dissipation by ferrous ion impurities, following the model developed by Galt, yields a magnetic susceptibility that may exceed by factors of  $10^5$  or more the values previously assumed for interstellar ferromagnetic grains. For large, multi-domain grains the dissipation associated with domain wall motion gives about the same range under interstellar conditions as for single domains. If iron atoms in a grain are gathered in clumps of about 100 atoms each, the grain is "super-paramagnetic" and  $\chi''$  can be even greater than for a ferromagnetic grain. The values of  $B$  required for grain orientation with these ferromagnetic and super-paramagnetic grains are between  $10^{-6}$  and  $10^{-8}$  gauss. We conclude that orientation of interstellar grains in fields as low as  $10^{-6}$  gauss seems a not-implausible expectation.

## I. INTRODUCTION

The polarization of starlight is generally assumed to be produced by alignment of elongated or anisotropic grains in an interstellar magnetic field. If one assumes that the grains are permanent magnets, oriented according to the usual Langevin formula, then according to Spitzer and Tukey (1951) a magnetic field,  $B$ , greater than  $10^{-4}$  gauss is required for appreciable orientation even if each grain is a single ferromagnetic domain. So high a value for  $B$  is quite incompatible with our knowledge of the interstellar gas, in particular with the observed motions in the gas. According to a theory proposed by Davis and Greenstein (1951; referred to here as "DG") grains can be oriented by a magnetic relaxation effect; if the magnetic susceptibility has an imaginary part, these authors showed that, provided disorienting collisions with gas atoms are sufficiently rare, the steady magnetic torque would tend to orient elongated grains so that they would be spinning about their transverse axes and the rotation axes would in turn be spinning about their transverse axes and the rotation axes would in turn be oriented parallel to the magnetic field. With reasonable estimates for paramagnetic relaxation,

DG concluded that a magnetic field between  $10^{-5}$  and  $10^{-4}$  gauss could produce enough alignment to explain the observed polarization.

A magnetic field somewhat greater than  $10^{-5}$  gauss is consistent with some theoretical expectations (Woltjer 1962), but there is some reason to believe that  $B$  may be more nearly in the neighborhood of  $10^{-6}$  gauss (Spitzer 1962). In an attempt to explain polarization at much lower fields, Henry (1958) extended the DG analysis to include ferromagnetic relaxation; Spitzer and Tukey (1951) had argued that ferromagnetic grains might well be expected in view of the selective evaporation of the more volatile materials and the not-improbable formation of such ferromagnetic compounds as  $\text{Fe}_2\text{O}_3$ ,  $\text{Fe}_3\text{O}_4$ , and  $\text{MgFe}_2\text{O}_4$ . Henry concluded that ferromagnetic grains could be oriented even by fields as weak as  $10^{-7}$  gauss.

Somewhat comparable results have been obtained for graphite flakes, which have been proposed by Cayrel and Schatzman (1954) as possible agents for producing the interstellar polarization; the optical anisotropy in graphite is so large that only very partial orientation would be required. The magnetic field required to explain the polarization by this mechanism was found to be between  $10^{-4}$  and  $10^{-5}$  gauss, according to Cayrel and Schatzman. However, a more recent evaluation by Wickramasinghe (1962) of the magnetic susceptibility of graphite asserts that the weak alignment required could readily be achieved in a galactic magnetic field of only  $10^{-6}$  gauss. On the basis of these results, it appeared that magnetic alignment of interstellar grains appeared a reasonable explanation of interstellar polarization, and that with either ferromagnetic or graphite particles even the lowest field estimates would provide adequate alignment.

This apparently satisfactory state ran into two separate difficulties. The basic mechanism of orientation proposed by DG was called into question by Kittel (1956), who pointed out that thermal fluctuations of magnetization within the grains should provide a disorienting mechanism. This effect, which must produce a completely isotropic distribution in thermodynamic equilibrium, was ignored in the DG analysis. The possibility arose that under conditions in interstellar space this inverse process would prevent magnetic relaxation from producing any appreciable orientation.

A second difficulty was pointed out by Cugnon (1963), who repeated and extended the analysis by Henry and in particular corrected a large numerical error in Henry's calculations. According to the corrected results, orientation of ferromagnetic domains requires a field of about  $10^{-5}$  gauss, not much less than the value needed for orientation of paramagnetic grains. About the same value was found to be required for orientation of graphite particles. On the assumption that magnetic orientation was operative at all under interstellar conditions, the magnetic field required might still be consistent with the lower theoretical estimates if graphite particles were responsible for the polarization, but paramagnetic or ferromagnetic particles would require relatively high fields.

The present paper is devoted to a discussion of these two problems with magnetic orientation. Section II considers the magnetic orientation of grains when inverse disorienting processes are properly taken into account. The net result of the analysis is not to improve significantly on the orientation estimates by DG, since only idealized limiting uses have been considered. Instead, the discussion provides a different and in some ways more realistic physical picture, which brings out more clearly the role played by thermal fluctuations of magnetization. In § III the rate of magnetic relaxation in a rotating ferromagnetic particle is reviewed, on the basis of additional theoretical and experimental knowledge obtained during the last 10 years. These estimates indicate a reasonable probability that ferromagnetic interstellar grains can be oriented appreciably by a magnetic field as weak as  $10^{-6}$  gauss.

## II. THEORY OF STEADY-STATE MAGNETIC ALIGNMENT

We consider an assembly of identical spheroids rotating in a uniform magnetic field,  $B$ . The spheroids are assumed to be characterized by an internal temperature,  $T_i$ , while

the atoms in the surrounding gas are assumed to have a Maxwellian distribution of velocities at the gas temperature,  $T_g$ . The rotational momentum and orientation of the spheroids will be changed by collisions with the gas atoms and by magnetic torques, including the fluctuating torques associated with thermal fluctuations of the magnetization. Evidently many collisions and prolonged thermal fluctuations will be required to produce appreciable changes in the macroscopic momentum of a spheroid. The change in the distribution of the spheroids in phase space under these conditions is given by the Fokker-Planck equation, discussed in general by Chandrasekhar (1943) and given in detailed form by Rosenbluth, MacDonald, and Judd (1957).

We apply this equation here to spherical particles, composed of a homogeneous isotropic material. Alignment of non-spherical particles is then discussed on the basis of an approximate, idealized model.

#### a) Spherical Grains

For spherical grains with isotropic properties the analysis becomes very simple and an exact solution for the distribution function is possible. Let  $J_x$ ,  $J_y$ , and  $J_z$  represent the angular momenta of a sphere about each of the three coordinate axes, with the  $z$ -axis taken to be parallel to the magnetic field,  $\mathbf{B}$ . Let  $f_x(J_x)dJ_x$  represent the fraction of the spheres for which the angular momentum about the  $x$ -axis is between  $J_x$  and  $J_x + dJ_x$ . Then the Fokker-Planck equation becomes

$$\frac{\partial f_x}{\partial t} = -\frac{\partial}{\partial J_x}(f_x \langle \Delta J_x \rangle) + \frac{1}{2} \frac{\partial^2}{\partial J_x^2} [f_x \langle (\Delta J_x)^2 \rangle], \quad (1)$$

with corresponding equations for  $f_y(J_y)$  and  $f_z(J_z)$ . The quantity  $\langle \Delta J_x \rangle$  represents the mean change of  $J_x$  per unit time, while  $\langle (\Delta J_x)^2 \rangle$  represents the mean value of  $(\Delta J_x)^2$  per unit time. These quantities are sometimes called "diffusion coefficients." We denote by  $\langle \Delta J_x \rangle_g$  and  $\langle \Delta J_x \rangle_i$  the values of this diffusion coefficient resulting from collisions with the gas and from magnetic torques, respectively, and similarly for the other diffusion coefficients also.

The values of  $\langle \Delta J_x \rangle_g$  and  $\langle (\Delta J_x)^2 \rangle_g$  may be computed readily. The number of atomic collisions per unit time with a sphere of radius  $a$  equals  $\pi a^2 n v$ , where  $n$  is the number of atoms per cubic centimeter and  $v$  is the mean atomic velocity; since the solid particle is much more massive than an atom, its translational velocity is much less in kinetic equilibrium and will be ignored. The mean angular momentum of the atoms striking a sphere is zero, but when the particles leave, they will carry with them an angular momentum proportional to the rotational velocity of the sphere. The simplest assumption is that on the average the atoms leave with the same angular momentum, per unit mass, as the surface of the rotating sphere, for which  $dJ_x/dm$  equals  $\frac{2}{3}\omega_x a^2$ , where  $\omega_x$  is the angular velocity about the  $x$ -axis. Other assumptions would be equally possible and would give somewhat different numerical results. On this assumption, we have

$$\langle \Delta J_x \rangle_g = -\frac{2}{3}\pi\omega_x a^4 n m v. \quad (2)$$

Since

$$J_x = I\omega_x, \quad (3)$$

where  $I$  is the moment of inertia of the sphere about a diameter, equation (2) may be written, if we express the mean atom velocity  $v$  in terms of the gas temperature  $T_g$ ,

$$\langle \Delta J_x \rangle_g = 4(2\phi m k T_g)^{1/2} a^4 n \frac{J_x}{3I}. \quad (4)$$

The dominant contribution to  $\langle (\Delta J_x)^2 \rangle_g$  is independent of  $\omega_x$ , but arises from the angular momentum given up by each atom on impact with the sphere, and taken away

on leaving. If a gas atom hits the sphere at a projected distance  $r$  from the center, the angular momentum  $\Delta J$  conveyed to the sphere will be  $mvr$ . Since the mean square value of  $r$  is  $\frac{1}{2}a^2$ , we have immediately

$$\langle(\Delta J^2)\rangle_g = \pi a^2 n m^2 \langle v^3 \rangle a^2, \quad (5)$$

where now  $\langle v^3 \rangle$  represents the mean value of  $v^3$ , and where we have multiplied the right-hand side by 2 to take account of the corresponding contribution to  $(\Delta J)^2$  when the atoms leave the particle. If we evaluate the mean value of  $v^3$  for a Maxwell-Boltzmann distribution, and let  $\langle(\Delta J_x^2)\rangle_g$  equal  $\frac{1}{3}\langle(\Delta J)^2\rangle_g$  from symmetry, we find

$$\langle(\Delta J_x^2)\rangle_g = \frac{8}{3}kT(2\pi mkT_g)^{1/2}a^4n. \quad (6)$$

If for the moment we neglect the diffusion coefficients arising from magnetic torques, we may insert equations (4) and (6) into equation (1). We find that  $\partial f_x/\partial t$  vanishes if

$$\log f_x(J_x) = -\frac{J_x^2}{2IkT_g} + \text{const.}; \quad (7)$$

i.e.,  $f_x$  is constant if the distribution function for  $J_x$  is Maxwellian at the same temperature as the gas. Evidently this result is required thermodynamically, and yields a general relation between the diffusion coefficients. For a process which is produced by interactions characterized by some temperature  $T$ , this relation becomes

$$\langle\Delta J_x\rangle = \frac{\partial}{\partial J_x}\langle(\Delta J_x)^2\rangle - \frac{J_x}{2IkT_g}\langle(\Delta J_x)^2\rangle. \quad (8)$$

Evidently equations (4) and (6) satisfy this general relation.

When magnetic interactions are taken into account, additional diffusion coefficients appear. From equation (81) of DG it is readily shown that for a sphere ( $\gamma$  equal to unity)

$$\frac{\langle\Delta J_x\rangle_i}{J_x} = \frac{\langle\Delta J_y\rangle_i}{J_y} = -\frac{VB^2\chi''}{I\omega}, \quad (9)$$

where  $V$  is the volume of the sphere,  $\chi''$  is the imaginary part of the magnetic susceptibility, and  $\omega$  is the total angular velocity, equal to  $J/I$ . For paramagnetic materials  $\chi''/\omega$  may be assumed constant for conditions in interstellar space, and hence  $\langle\Delta J_x\rangle_i/J_x$  and  $\langle\Delta J_y\rangle_i/J_y$  are constants for any one sphere. From equation (8) we may therefore write

$$\langle(\Delta J_x)^2\rangle_i = \langle(J_y)^2\rangle_i = \frac{2kTVB^2\chi''}{\omega}. \quad (10)$$

Equation (10) gives the diffusion coefficients resulting from the magnetic torques on thermal fluctuations of the internal magnetic field. The angular momentum about the  $z$ -axis is evidently unaffected by the magnetic field parallel to this axis, and we have

$$\langle\Delta J_z\rangle_i = \langle(\Delta J_z)^2\rangle_i = 0. \quad (11)$$

When the diffusion coefficients are given by the sums of the coefficients for gas collisions and for magnetic interactions, equation (1) yields a simple solution for the steady state, in which  $\partial f/\partial t$  vanishes. For  $f_z$ , we need consider only gas collisions, because of equation (11), and an equation similar to (7) is valid, giving

$$f_z(J_z) = (2\pi IkT_g)^{-1/2} \exp\left(-\frac{J_z^2}{2IkT_g}\right), \quad (12)$$

where the constant of normalization has been adjusted so that the integral of  $f_z$  over all  $J_z$  equals unity. For  $f_x$  we obtain, after some algebra, the steady-state solution

$$f_x(J_x) = (2\pi I k T_{av})^{-1/2} \exp \left\{ -\frac{J_x^2}{2 I k T_{av}} \right\}, \quad (13)$$

where  $T_{av}$  is defined by

$$T_{av} = \frac{T_g + \delta T_i}{1 + \delta}, \quad \delta \equiv \frac{\langle \Delta J_x \rangle_i}{\langle \Delta J_x \rangle_g} = \frac{\chi'' B^2}{2 a \omega n} \left( \frac{2\pi}{m k T_g} \right)^{1/2}. \quad (14)$$

Evidently the distribution of angular momenta about each separate axis is Maxwellian but with different effective temperatures about different axes. For the two axes perpendicular to  $B$  the effective temperature is nearly equal to the gas temperature,  $T_g$ , when  $\delta$  is small (corresponding to  $B$  small or  $n$  large), but approaches  $T_i$ , the internal temperature of the spheroids, when  $\delta$  is large and magnetic interactions dominate over collisions with the gas. Thus the effective rotational temperature about these two axes varies between  $T_g$  and  $T_i$ , depending on the relative strengths of the two relevant interactions.

We may integrate these expressions over  $J$  to compute the distribution of angular-momentum vectors over different directions. Let us denote by  $f_\beta(\beta)d\Omega$  the fraction of spheres whose angular-momentum vectors lie within a solid angle  $d\Omega$  about some particular direction, at an angle  $\beta$  to  $B$ ; evidently  $f_\beta$  depends only on  $\beta$  and is independent of the azimuthal angle about the  $z$ -axis. With straightforward algebra, we obtain

$$f_\beta(\beta) = \frac{1}{4\pi} \left( \frac{T_{av}}{T_g} \right)^{1/2} \frac{1}{[(T_{av}/T_g) \cos^2 \beta + \sin^2 \beta]^{3/2}}. \quad (15)$$

It may be noted that if, following DG, we defined the angular distribution function  $f_\beta(\beta)$  as the particle density per unit angle instead of per unit solid angle, an additional factor  $2\pi \sin \beta$  would appear in  $f_\beta(\beta)$ . Evidently if  $T_{av}/T_g$  is small,  $f_\beta(\beta)$  in equation (15) will be sharply peaked around zero  $\beta$ . This result corresponds to the higher effective rotational temperature for  $\beta = 0$  than for  $\beta = \pi/2$  in this situation, giving most of the rotational energy about an axis parallel to  $B$ .

### b) Spheroidal Grains

We turn next to the more general problem of spheroidal particles, each with a moment of inertia  $I$  about the axis of symmetry and  $\gamma I$  about each of the two transverse axes. In principle, the computation of diffusion coefficients given for spherical grains can also be carried through for spheroidal grains, and the exact steady-state distribution determined. This computation would be required for a precise determination of the polarization produced by grains of some particular type. At the present time there are so many uncertainties in the theoretical explanation of interstellar polarization that such precise computations do not seem required. Instead, we shall carry out a much simplified analysis, based on an idealized model.

First we consider the distribution of angular momenta in the absence of any external torques other than those due to collisions. The distribution functions will then all be Maxwellian at the gas temperature,  $T_g$ . If we let  $f_A(J_A)dJ_A$  be the fraction of the spheroids whose angular momentum about the symmetry axis of each spheroid lies between  $J_A$  and  $J_A + dJ_A$ , then the functional form of  $f_A$  will be identical to that of  $f_z$  given in equation (12). Similarly, if we define  $f_B(J_B)$  in terms of the angular momentum  $J_B$  about either of the transverse axes, then  $f_B$  is also given by equation (12) but with  $\gamma I$  replacing  $I$ . We define  $f_\theta(\theta)d\Omega$  as the fraction of the spheroids for which the principal axis lies within a range of solid angle,  $d\Omega$ , about some direction at an angle  $\theta$  with respect to  $J$ . The

evaluation of  $f_\theta$  from  $f_A$  and  $f_B$  proceeds exactly as the evaluation of  $f_\beta$  from  $f_z$  and  $f_x$ , and we obtain

$$f_\theta(\theta) = \frac{\gamma^{1/2}}{4\pi} \frac{1}{(\gamma^2 \cos^2 \theta + \sin^2 \theta)^{3/2}}, \quad (16)$$

a result equivalent to that in equation (37) of DG.

Equations (15) and (16), and the derivation of these relationships here, bring out the physical reason for alignment of spheroids in a magnetic field. In a steady state the distribution of rotational kinetic energy in a fixed direction will tend toward a Maxwellian distribution at a temperature which is intermediate between the two temperatures characterizing the problem, the internal temperature,  $T_i$ , of the material within the spheroid, and the kinetic temperature,  $T_g$ , of the gas atoms which collide with the spheroid. This effective temperature, which we denote by  $T_e$ , will vary, of course, with  $\beta$ . For some models  $T_e$  will depend on the absolute magnitude of  $J$ , in which case the distribution of  $J$  in a fixed direction will presumably not be Maxwellian. In the simple case that  $\chi''/\omega$  is a constant, the diffusion coefficients  $\langle(\Delta J_i)^2\rangle$  will be independent of the magnitude of  $J$ , and  $T_e$  should also be independent of  $J$ . If we make the simplifying assumption that  $T_e$  is independent also of  $\theta$ , the angle between the spheroidal axes of symmetry and  $J$ , then the distributions of  $\beta$  and  $\theta$  become uncoupled. In this special case we may write

$$f(\beta, \theta) = f_\beta(\beta)f_\theta(\theta), \quad (17)$$

where  $f_\beta(\beta)$  and  $f_\theta(\theta)$  are given in equations (15) and (16), respectively.

While equation (17) should represent the general features of the problem, it cannot be correct in detail (we are indebted to Dr. L. Davis for pointing out this result). For a spheroid in a strong magnetic field (large  $\delta$ )  $T_e$  for  $\beta$  equal to zero depend upon  $\theta$ ; for  $\theta$  equal to zero in this case, there is no coupling with the magnetic field or with  $T_i$ , while for finite  $\theta$  such coupling must appear because of the nutation of the spheroid. For low  $\delta$  or  $\gamma$ , however, equation (17) is probably not far from the truth, and we shall adopt it here to obtain a rough indication of the alignment to be expected.

We now apply equation (17) to a quantitative evaluation of the alignment, using the quantity  $F$  introduced by DG. If  $\alpha_z$  is the angle between the symmetry axis of the spheroid and the magnetic field  $B$ , then  $F$  is defined as the difference between the average value of  $\cos^2 \alpha_z$  for an isotropic distribution and the corresponding average for the actual distribution. From equation (109) of DG, we have

$$F = \frac{1}{3} - 4\pi^2 \int_0^\pi \int (\cos^2 \beta \cos^2 \theta + \frac{1}{2} \sin^2 \beta \sin^2 \theta) f(\beta, \theta) \sin \beta \sin \theta d\beta d\theta. \quad (18)$$

If now we insert equation (17) into (18), substituting from equations (15) and (16), we obtain, after some algebra,

$$F = -\frac{3}{2} q \left( \frac{T_{av}}{T_g} - 1 \right) q(\gamma - 1), \quad (19)$$

where

$$q(x) \equiv -\frac{1}{3} + \frac{1}{x} \left[ \left( \frac{1+x}{x} \right)^{1/2} \sinh^{-1} x^{1/2} - 1 \right]. \quad (20)$$

When  $x$  is negative,  $(-x)^{-1/2} \sin^{-1}(-x)^{1/2}$  may be substituted for  $x^{-1/2} \sinh^{-1} x^{1/2}$  in equation (20). The function  $q(x)$  has the following limiting forms:

$$q(x) = -\frac{2}{15} x \quad (x \ll 1), \quad q(-1) = \frac{2}{3}, \quad q(\infty) = -\frac{1}{3}. \quad (21)$$

Values of  $q(x)$  for intermediate  $x$  are given in Table 1. If  $T_{av}$  is zero, one would expect that  $F$  should equal  $\frac{1}{3}$ , the limiting value for complete orientation of prolate spheroids,

resulting when  $T_i$  is much less than  $T_g$  and collisions are relatively unimportant. Equation (19) yields the result only if  $\gamma$  is large, a result of the incorrect asymptotic behavior of equation (17) noted above. For weak alignment, when  $T_{av}$  is nearly equal to  $T_g$ , and  $\gamma$  is only slightly greater than unity, we may use the linear expansion of  $q(x)$  in the first equation of (21), and, substituting from equations (14) for  $T_{av}$  and  $\delta$ , we obtain

$$F = \frac{\chi'' B^2}{75 a \omega n} \left( \frac{2\pi}{m k T_g} \right)^{1/2} (\gamma - 1) \left( 1 - \frac{T_i}{T_g} \right), \quad (22)$$

where we have replaced  $I$  by  $2Ma^2/5$ . Equation (22) gives a value less than that given by DG's equations (90), (91), (15), and (16) by a factor  $9(6\pi)^{1/2}(1 - T_i/T_g)/80$  or  $0.488(1 - T_i/T_g)$ . As expected, the orientation disappears if  $T_i$  equals  $T_g$  and reverses sign if  $T_i$  exceeds  $T_g$ .

Evidently the analysis presented in this section gives an orientation which does not differ substantially from that found by DG, provided that the internal temperature,  $T_i$ , of the spheroid is much less than the gas temperature,  $T_g$ . While no great confidence

TABLE 1  
VALUES OF THE FUNCTION  $q(x)$

| $x$     | $q(x)$ | $x$     | $q(x)$ | $x$     | $q(x)$ | $x$   | $q(x)$ | $x$  | $q(x)$ |
|---------|--------|---------|--------|---------|--------|-------|--------|------|--------|
| -1 0.   | 0 667  | 0 0 ... | 0 000  | 1 0 ... | -0 087 | 3 0 . | -0.160 | 15.  | -0 258 |
| -0 9    | 315    | 1       | -.013  | 1 2 ... | -.098  | 3 5   | -.171  | 20.. | -.270  |
| -0 8    | .225   | .2 .    | -.024  | 1 4 .   | -.108  | 4 0   | -.180  | 30 . | -.286  |
| -0 7 .  | 168    | 3 .     | -.034  | 1 6 ... | -.116  | 4 5   | -.188  | 40   | -.294  |
| -0 6 .  | .128   | .4....  | -.044  | 1 8 .   | -.124  | 5 0 . | -.195  | 50   | -.300  |
| -0 5    | 096    | 5       | -.052  | 2 0 .   | -.131  | 6 0 . | -.207  | 60 . | -.304  |
| -0 4 .  | .070   | 6 ..    | -.060  | 2 2 ..  | -.138  | 7 0   | -.217  | 70   | -.307  |
| -0 3 .. | 049    | 7       | -.068  | 2 4 ..  | -.144  | 8 0   | -.225  | 80   | -.310  |
| -0 2 .  | 031    | .8 ..   | -.074  | 2 6 .   | -.150  | 9 0   | -.231  | 90   | -.312  |
| -0 1    | 0 014  | 0 9 .   | -0 081 | 2 8 .   | -0 155 | 10 0  | -0 237 | 100  | -0 313 |

can be placed in the numerical coefficient in equation (22), the physical basis for the orientation is somewhat different in detail from that presented by DG and more consistent with thermodynamic arguments. However, it would appear to validate the DG result. We may assume that for spheroids composed of anisotropic material, such as single ferromagnetic domains and graphite particles, the results obtained by Cugnon, extending the analysis of DG, are also approximately valid, again with the condition that  $T_i$  be much smaller than  $T_g$ .

### III. A REVIEW OF MAGNETIC-RELAXATION MECHANISMS

As mentioned earlier, a number of different models of magnetic relaxation have been proposed to account for the magnitude of the grain alignment. In this section we explore some of these models in more detail and attempt to improve or set bounds on the various estimates of the parameter  $\chi''/\omega$  which appears in the previous section (see the second of eqs. [14] and [22]) and which we here denote by  $K$ . Recent years have seen a good deal of progress in the understanding of magnetic-relaxation processes, and we hope to capitalize on this improved understanding in our re-estimates. Unfortunately, we are forced to use mainly theoretical insights or extrapolations, since there are relatively few experimental results which may be applied directly to the conditions of the interstellar dust problem. In order to define the magnetic quantities of importance and to specify the assumptions of the models, we shall first re-examine the general analysis of the dynamics of a rotating spheroidal particle, first presented by Henry (1958) and by Cugnon (1963). Following

this, we shall treat separately the behavior of paramagnetic and ferromagnetic particles. We are not prepared at this time to make any definitive comments on the relaxation behavior of diamagnetic graphite particles (Hoyle and Wickramasinghe 1962).

a) *Dynamics of Magnetic Relaxation*

It was seen in § II that the Henry-Cugnon analysis would be expected to lead to results consistent with the more rigorous statistical mechanical approach. We have extended this method of analysis to include two features of a general model of magnetic relaxation: viz., (i) a tensor susceptibility, and (ii) an arbitrary dispersive character for the susceptibility. Following Cugnon, we have derived the equations of motion of an axial symmetric particle assuming that the symmetry axis is also an axis of magnetic symmetry. Referring the induced magnetization to a system of coordinates fixed in the rotating particle, we may write quite generally

$$\begin{aligned} M_x(t) &= \int_0^{+\infty} \{ \chi_{xx}(\tau) H_x(t-\tau) + \chi_{xy}(\tau) H_y(t-\tau) \} d\tau, \\ M_y(t) &= \int_0^{+\infty} \{ \chi_{xx}(\tau) H_y(t-\tau) - \chi_{xy}(\tau) H_x(t-\tau) \} d\tau, \\ M_z(t) &= \int_0^{+\infty} \chi_{zz}(\tau) H_z(t-\tau) d\tau. \end{aligned} \quad (23)$$

Again following the Henry-Cugnon procedure, we have derived expressions for the average values of the relaxation torque  $\langle L \rangle$  and the change in rotational kinetic energy  $\langle dT/dt \rangle$ . In Cugnon's notation these expressions may be written

$$\langle L \rangle = \frac{V}{4J} [ (J \times H) \times H ] \quad (24)$$

$$\times [ (1 + \cos \theta)^2 \chi''_-(\omega_1 + \omega_2) - (1 - \cos \theta)^2 \chi''_-(\omega_1 - \omega_2) + 2 \sin^2 \theta \chi''_z(\omega_2) ],$$

$$\begin{aligned} \left\langle \frac{dT}{dt} \right\rangle &= - (V \omega_2 H^2) [ \sin^2 \theta \cos \phi (\gamma - 1) \chi''_-(\omega_1) \\ &+ \sin^2 \beta \{ \frac{1}{4} (1 + \cos \theta) \chi''_-(\omega_1 + \omega_2) [ \gamma \cos \theta (1 + \cos \theta) + \sin^2 \theta ] \\ &+ \frac{1}{4} (1 - \cos \theta) \chi''_-(\omega_1 - \omega_2) [ \gamma \cos \theta (1 - \cos \theta) - \sin^2 \theta ] \\ &+ \frac{1}{2} \sin^2 \theta \chi''_z(\omega_2) - \sin^2 \theta \cos \theta (\gamma - 1) \chi''_-(\omega_1) \} ], \end{aligned} \quad (25)$$

where

$$\chi(\omega) = \chi'(\omega) - i\chi''(\omega) = \int_0^{+\infty} \chi(\tau) e^{i\omega\tau} d\tau, \quad \chi_-(\omega) = \chi_{xx}(\omega) + i\chi_{xy}(\omega),$$

and

$$\omega_1 = \left( \frac{\gamma - 1}{\gamma} \right) \omega_z, \quad \omega_2 = \left( \frac{1}{\gamma \cos \theta} \right) \omega_z.$$

As we shall see later, at very low frequencies most relaxation models predict that  $\chi''$  is proportional to frequency. In this approximation, equations (24) and (25) reduce to Cugnon's equations (12) and (13) where now, however, we must interpret  $K_T$  as  $\chi''_-(\omega)/\omega$  and  $K_A$  as  $\chi''_z(\omega)/\omega$ . We shall see later for the case of ferromagnetic particles that this reinterpretation has some important consequences.



## b) Paramagnetic Relaxation

In general, the power dissipation in a paramagnetic system is usually discussed in terms of the two distinct mechanisms of spin-lattice and spin-spin relaxation (see Gorter 1947). In their estimate of interstellar-grain alignment, DG implicitly omit any consideration of the spin-lattice process. Although we accept the basic correctness of their view, it is perhaps useful to examine briefly the *justification* for neglecting this normally dominant absorption mechanism. According to the famous phenomenological model of Casimir and DuPré (1938), spin-lattice absorption is treated by invoking the supposition that the sample may be divided into two interacting thermodynamic systems, the coupled system of electronic spins and the crystalline lattice which is in strong contact with a thermal reservoir. The interaction between the systems is characterized by a time constant  $\tau_{SL}$  which is a measure of the time required for the spin system to come to equilibrium with the lattice. A dispersion in the paramagnetic susceptibility occurs when the modulation frequency  $\omega$  of an applied magnetic field becomes comparable to  $\tau_{SL}^{-1}$ . By thermodynamic arguments, Casimir and DuPré showed that the spin-lattice contribution to the differential susceptibility is given by

$$\chi_{SL}(\omega) = \chi_0(H) \left[ \frac{C_M}{C_H} + \left( \frac{C_H - C_M}{C_H} \right) \left( \frac{1}{1 + i\omega\tau_{SL}} \right) \right], \quad (26)$$

where  $\chi_0(H)$ ,  $C_M$ , and  $C_H$  are, respectively, the isothermal susceptibility at a bias field  $H$ , the magnetic specific heat at constant magnetization, and the specific heat at constant field. It may also be shown (Caspers 1964) that

$$C_H - C_M = T \left( \frac{\partial M}{\partial T} \right)_H^2 \left( \frac{\partial M}{\partial H} \right)_T^{-1}. \quad (27)$$

Thus, for any paramagnetic system where the time-averaged magnetic moment is zero (i.e., the bias field  $H$  is zero),  $C_H$  and  $C_M$  are identical, and  $\chi''_{SL}(\omega)$  vanishes. From this argument, we may safely agree with DG that the spin-lattice process plays no role in the paramagnetic relaxation of interstellar dust particles since these particles would have no net moment in the rotating coordinate system.

Thus, the dispersion in the susceptibility at zero biasing field results from a loss of internal equilibrium in the spin system. The spin-spin relaxation is usually treated by Van Vleck's method of moments (Van Vleck 1948; Caspers 1964). By this method, the various moments of the line-shape function  $\chi''(\omega)$  may be evaluated exactly for any given set of interactions within the spin system. However, the calculation of these moments is an extremely tedious operation, and so interpolating line-shape functions have been evolved to fit various models of the dominant interactions within the spin system. If only magnetic dipole-dipole interactions are considered, Broer (1943) found that the correct theoretical fourth to squared second moment ratio is approximately satisfied by a Gaussian function,

$$\chi''_B(\omega) = \chi(0) \left( \frac{\pi}{2} \right)^{1/2} \left( \frac{\omega}{\langle \omega^2 \rangle^{1/2}} \right) \exp \left( - \frac{\omega^2}{2 \langle \omega^2 \rangle} \right), \quad (28)$$

where  $\chi(0)$  and  $\langle \omega^2 \rangle$  are, respectively, the static susceptibility and the second moment of the line-shape function. It is this function that DG use in their estimate. Locher and Gorter (1961) have proposed a more general line-shape function of the form

$$\chi''_{LG}(\omega) = \chi(0) \left( \frac{\pi}{2} \right)^{1/2} [a(\pi)^{1/2} \exp(a^2) \operatorname{Erfc} a]^{-1} \left( \frac{2^{1/2} a \tau \omega}{1 + \omega^2 \tau^2} \right) \exp(-a^2 \tau^2 \omega^2), \quad (29)$$

where the parameter  $\alpha$  is a function of the moments ratio,  $\langle \omega^4 \rangle / \langle \omega^2 \rangle^2$ , and measures the effectiveness of other interactions such as exchange, hyperfine, and crystal field. In the limit of  $\alpha \rightarrow \infty$ ,  $\chi''_{LG}$  reduces to the Gaussian  $\chi''_B$  with  $\alpha^2 \tau^2 = (2\langle \omega^2 \rangle)^{-1}$ . An exchange-dominated relaxation process is represented by the limit  $\alpha = 0$ , in which  $\chi''_{LG}$  reduces to a simple Debye function where  $1/\tau$  is the "exchange rate." Equation (29) has been used quite successfully by Locher and Gorter (1961) to interpret the relaxation spectra of a series of copper Tutton salts where the dipole-dipole and exchange interactions have varying degrees of relative importance. For our problem, we are primarily concerned with obtaining a reasonable estimate of the low-frequency limit of the parameter  $K = \chi''(\omega)/\omega$ . In Figure 1 we have plotted as a function of  $\alpha$  the ratio  $R$  of the low-frequency limits of the two possible line-shape functions, i.e.,

$$R = \lim_{\omega \rightarrow 0} [\chi''_{LG}(\omega) / \omega] / \lim_{\omega \rightarrow 0} [\chi''_B(\omega) / \omega].$$

It is apparent from this figure that  $R$  lies in the range between 1 and 10 for any reasonable assumption about the value of  $\alpha$ . Thus, in the analysis of DG, any error arising from the use of equation (28) will certainly not exceed one order of magnitude and the Gaussian would, in fact, underestimate the relaxation.

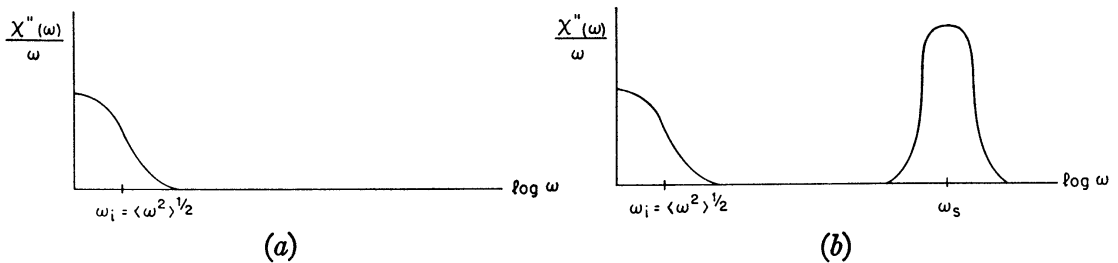


FIG. 1.—The ratio

$$R = \lim_{\omega \rightarrow 0} [\chi''_{LG}(\omega) / \omega] / \lim_{\omega \rightarrow 0} [\chi''_B(\omega) / \omega]$$

as a function of the moments ratio parameter  $\alpha$ .

Before turning to appropriate estimates of  $\langle \omega^2 \rangle$ , we should say a word about the effect of local Stark or crystalline fields on the line shape of the relaxation spectrum. In Figure 2 we have sketched, for contrast, the approximate frequency behavior, at zero bias field, of  $\chi''(\omega)/\omega$  for a system where the Stark interaction is (a) negligible (say, the case of  $\text{Fe}^{2+}$  ion), and (b) important (say, the case of  $\text{Fe}^{3+}$  ion). The crystal field leads to additional broad absorption peaks near  $\omega_s = (\text{Stark splitting})/\hbar$ . However, for  $\text{Fe}^{3+}$  ion  $\omega_s$  would be in the microwave range and the additional absorption peaks would probably have no effect on the dust alignment problem.

In assessing a reasonable value of the second moment,  $\langle \omega^2 \rangle$ , in equation (28), we must consider both the dipole-dipole and possible hyperfine interactions. We shall neglect the effect (as did DG) of Stark fields on  $\langle \omega^2 \rangle$ . As we have noted in the previous paragraph, the Stark field components of  $\langle \omega^2 \rangle$  contribute to high-frequency absorption peaks. The exchange interaction does not contribute to  $\langle \omega^2 \rangle$  and may be accounted for by some small modification in the  $\alpha$  parameter above. Following Caspers (1964), we may write

$$\langle \omega^2 \rangle = \left( \frac{g\mu_B}{\hbar} \right)^2 \left( H_{dd}^2 + \frac{1}{3} H_{hfs}^2 \right), \tag{30}$$

where  $H_{dd}^2 = 2S(S + 1) (g\mu_B)^2 n^2 \eta^2$  and  $H_{hfs}^2 = \frac{2}{3} I(I + 1) (A/g\mu_B)^2$ . In these expressions,  $n$  is the number of magnetic ions per unit volume,  $A$  is the average hyperfine interaction, and  $\eta^2$  is the lattice sum  $a^6 \sum_{i \neq j} r_{ij}^{-6}$  which takes on values between 7 and 9

for typical cases of interest. Combining equations (28) and (30), we have in the limit of low frequencies

$$K = \lim_{\omega \rightarrow 0} [\chi''_B(\omega)/\omega] = \frac{\sqrt{\pi}}{6} \left( \frac{\hbar}{kT} \right) \left[ \frac{S(S+1)}{\eta^2} \right]^{1/2} \left( 1 + \frac{H_{\text{hfs}}^2}{3H_{\text{dd}}^2} \right)^{-1/2}. \quad (31)$$

For trivalent iron ions

$$K \simeq (2.3 \times 10^{-12}) \frac{1}{T} \left( 1 + \frac{H_{\text{hfs}}^2}{3H_{\text{dd}}^2} \right)^{-1/2}, \quad (32)$$

which is in agreement with equation (68) of DG with the exception of the factor  $(1 + H_{\text{hfs}}^2/3H_{\text{dd}}^2)$ . The importance of this factor lies in the fact that  $H_{\text{hfs}}^2$  is essentially independent of concentration arising mainly from the dipolar interaction with the nuclear moments of neighboring hydrogen atoms. Thus,  $H_{\text{hfs}}^2$  sets a lower bound on  $\langle \omega^2 \rangle^{1/2}$  of about  $10^9$  rad/sec and at all concentrations of magnetic ions the exponential factor in equation (28) may be taken as unity for the typical rotational velocities ( $\sim 10^6$  rad/sec)

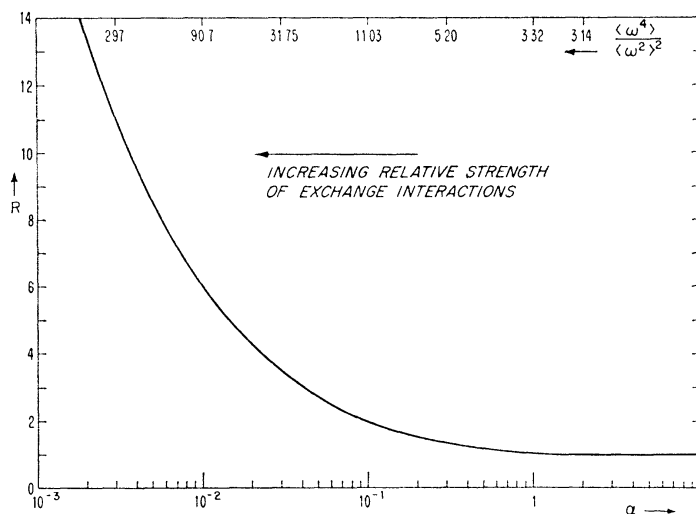


FIG. 2—Qualitative behavior of the parameter  $K = \chi''(\omega)/\omega$  for paramagnetic substances. (a) Case of no important crystalline field splittings, e.g.,  $\text{Fe}^{2+}$  ions. (b) Case of important crystalline field splittings,  $\text{Fe}^{3+}$  ions.

of the dust particles. However, in contrast with the conclusions of DG, the  $H_{\text{hfs}}^2$  contribution makes the  $K$ -value concentration-dependent. For the iron concentration assumed by DG, i.e., approximately 1 atomic per cent,  $H_{\text{dd}}^2$  would roughly equal  $H_{\text{hfs}}^2$  and their estimates would hold. Lower concentrations would lead to smaller values of  $K$ , so that equation (28) must be viewed as an *upper bound* for electronic paramagnetic relaxation process.

It is important to repeat here an extremely interesting suggestion of Purcell (1951). In the low-frequency limit equation (28) is not only insensitive to the concentration of magnetic species, but it is also independent of the magnitude of the individual magnetic moments! Purcell thus observes that *nuclear* paramagnetism should be as effective as electronic paramagnetism in relaxing the interstellar grains, if the particle rotational frequency is less than the second moment arising from the nuclear dipole-dipole interactions. From equation (30),  $\langle \omega^2 \rangle^{1/2}$  may be seen to be approximately  $2 \times 10^6$  rad/sec for a concentrated system of hydrogenic nuclear moments. Undoubtedly, almost all interstellar grains would have abundant concentrations of hydrogen and the  $K$ -value estimated by DG on the basis of equation (28) should hold even if no paramagnetic ions

are present in the grains. Hence, an approximate value of  $K = 10^{-12}$  (1/T) represents a lower bound for most interstellar grains independent of their particular compositions.

c) *Ferromagnetic Relaxation*

Spitzer and Tukey (1951) have described the kinetics of processes by which the magnetic ions in the interstellar particles may become concentrated to a point where magnetic ordering is possible. These processes might lead to totally ferromagnetic particles or to diamagnetic host particles with one or more imbedded fine ferromagnetic grains. In the subsequent discussions (Henry 1958) of the relaxation of such ferromagnetic particles, it has been assumed that magnetization of the particle is totally aligned, giving rise to a net permanent magnetic moment. However, before trying to estimate the magnitude of the ferromagnetic relaxation mechanisms, we must test this crucial assumption.

TABLE 2  
CRITICAL PARTICLE SIZE FOR SINGLE-DOMAIN BEHAVIOR

| Substance                                | Temperature<br>(° K) | Mean Critical<br>Diameter<br>(microns) | Reference* |
|--|----------------------|--|------------|
| Metallic iron . .                        | 4                    | 0.0230                                 | a          |
| Metallic iron . .                        | 207                  | 0.0218                                 | a          |
| Iron:cobalt (40:60)                      | 4                    | 0.0261                                 | a          |
| Iron:cobalt (40:60)                      | 207                  | 0.0284                                 | a          |
| NiFe <sub>2</sub> O <sub>4</sub> . . . . | 77-300               | ~0.030                                 | b          |
| NiFe <sub>2</sub> O <sub>4</sub>         | 300                  | <0.2                                   | c          |
| CoFe <sub>2</sub> O <sub>4</sub>         | 77-300               | ~0.070                                 | b          |
| CuFe <sub>2</sub> O <sub>4</sub>         | 77                   | 0.03-0.05                              | b          |
| γ-Fe <sub>2</sub> O <sub>3</sub>         |                      | ~1                                     | d          |
| CrO <sub>3</sub> . . . .                 |                      | ~0.3                                   | e          |

\* References: (a) E. F. Kneller and F. E. Laborsky, *J. Appl. Phys.*, **34**, 656 (1963); (b) A. E. Berkowitz and W. J. Schuele, *J. Appl. Phys.*, **30**, 134S (1959); (c) W. W. Malinofsky, R. W. Babbitt, and G. C. Sands, *J. Appl. Phys.*, **33**, 1206 (1962); (d) A. H. Morrish and L. A. K. Watt, *J. Appl. Phys.*, **29**, 1029 (1958); T. J. Swoboda, P. Arthur, Jr., N. L. Cox, J. N. Ingraham, A. L. Oppegard, and M. S. Sadler, *J. Appl. Phys.*, **32**, 374S (1961).

In 1930, Frankel and Dorfman first proposed that there should be a "critical particle" size below which single-domain behavior or total magnetic alignment is stable. The driving force for domain formation is, of course, the magnetostatic energy of the long-range dipolar interaction which opposes the dominant ordering tendencies of the short-range exchange interactions. For samples of ordinary size, a compromise between these competing interactions is found in multi-domain configurations of zero net moment which represent the lowest energy states of the system. However, in the limit of vanishing sample size, the exchange interaction must eventually inhibit the driving force for domain formation. The transition between single- and multi-domain behavior has been well established experimentally. In fact, the supposition of subcritical ferromagnetic particles is basic to the understanding of the whole technology of permanent magnets. However, as Brown (1959a) has discussed in detail, it has not been possible to derive theoretically a rigorous set of conditions for the boundaries of single-domain behavior. In view of the uncertainties in the theoretical situation, we appeal directly to experiment for information on the conditions for the transition. A number of observations of critical particle size are summarized in Table 2. Roughly speaking, for a group of quite dissimilar materials, the critical sizes lie in the range between 0.01 and 1.0  $\mu$ . However, particle diameters in the range 0.1-1.0  $\mu$  have been assumed usually for the interstellar

light-scattering centers. It thus appears that multi-domain dust particles might well be possible. Therefore, in the following paragraphs, we treat the ferromagnetic relaxation mechanisms of both single- and multi-domain systems.

1. *Relaxation of multi-domain particles.*—Useful guidance in estimating the susceptibility of ferromagnetic particles may be obtained from the considerable body of technical literature on the “initial” or small-signal permeabilities of a wide variety of magnetic materials. Three typical initial permeability frequency spectra are shown in Figure 3. As shown, it is common to observe two or, occasionally, three distinctly resolvable peaks in the frequency dependence of  $\mu''$ . Following the original suggestions of Snoek (1948), it is generally conceded that the upper peak is associated with a resonance in the magnetizing mechanism of “single-domain rotation.” This is the mechanism implicitly invoked by Henry (1958) which we will review in the following section. The lower-frequency peaks are generally attributed to the dynamics of the magnetizing mechanism of reversible domain-wall motion. Of course, these peaks occur only in multi-domain systems

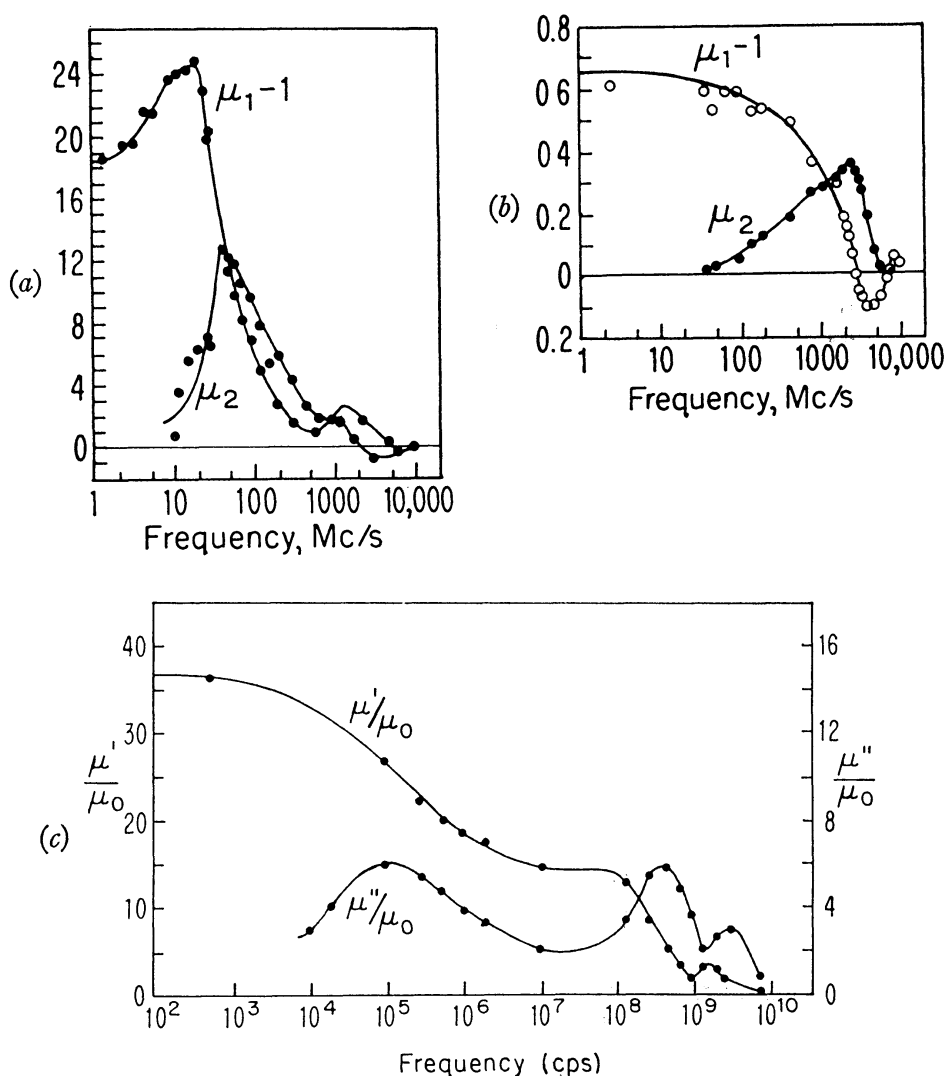


FIG. 3.—Initial permeability spectra of typical ferromagnetic insulators. (a) Sintered  $\text{MgFe}_2\text{O}_4$  at  $300^\circ\text{K}$  (Rado *et al.* 1956). (b) Single-domain  $\text{MgFe}_2\text{O}_4$  at  $300^\circ\text{K}$  (Rado *et al.* 1956). (c) Sintered  $\text{NiFe}_2\text{O}_4$  at  $150^\circ\text{K}$  (Epstein 1957).

(see Rado 1953; Rado, Wright, Emerson, and Tersis 1952; Rado, Folen, and Emerson 1956). We review the domain-wall loss mechanisms in this section.

The dynamics of reversible domain wall are usually treated in terms of a semi-phenomenological equation of motion for the displacement,  $z$ , of a plane domain wall. For a material of saturation magnetization  $M_s$ , subject to a small applied field  $H_A$ , the equation of motion for  $180^\circ$  walls may be expressed as (Kittel and Galt 1956)

$$m\ddot{z} + \beta\dot{z} + \alpha z = 2M_s H_A, \quad (33)$$

where  $m$ ,  $\beta$ , and  $\alpha$  are, respectively, the "effective mass," damping constant, and "stiffness" of the domain wall. The first or "effective mass" term in equation (33) arises from the inertial characteristics of the gyroscopic motion of the magnetic spins within the domain wall. The second term represents the effect of any mechanism which couples energy out of the coherent rotational motion of the magnetization within the wall. The third term of equation (33) is a first-order representation of the clamping effect on wall motion of local fluctuations in the domain-wall energy associated with inclusions, strains,

TABLE 3  
DOMAIN-WALL RELAXATION FREQUENCIES

| Substance  | $(\omega_{rel})_\infty = (\alpha/\beta)_\infty$<br>(rad/sec)                          | $\Delta E$ (eV)  | Reference* |
|--|---|--|------------|
| $\text{Cu}_{0.5}\text{Zn}_{0.5}\text{Fe}_2\text{O}_4$ . . . . .                              | $6 \times 10^{10}$  | 0 32   | <i>a</i>   |
| $\text{Ni}_{0.5}\text{Zn}_{0.5}\text{Fe}_2\text{O}_4$ . . . . .                              | $\sim 10^{13}$  | 40   | <i>a</i>   |
| $\text{Ni}_{0.5}\text{Zn}_{0.49}\text{Fe}^{\text{II}}_{0.02}\text{Fe}_2\text{O}_4$ . . . . . | $3 \times 10^9$   | .10  | <i>a</i>   |
| $\text{Ni}_{0.99}\text{Fe}^{\text{II}}_{0.01}\text{Fe}_2\text{O}_4$ . . . . .                | $4 \times 10^{10}$  | .10  | <i>a</i>   |
| $\text{Ni}_{0.85}\text{Fe}^{\text{II}}_{0.15}\text{Fe}_2\text{O}_4$ . . . . .                | $6 \times 10^{10}$  | 07   | <i>a</i>   |
| $\text{Ni}_{0.75}\text{Fe}^{\text{II}}_{0.25}\text{Fe}_2\text{O}_4$ . . . . .                | $3 \times 10^{12}$  | 055  | <i>b</i>   |
| $\text{Fe}_3\text{O}_4$ . . . . .  | $\left. \begin{array}{l} 4 \times 10^{12} \\ 6.7 \times 10^{23} \end{array} \right\}$ | $\left. \begin{array}{l} 055 \\ 0 36 \end{array} \right\}$ | <i>c</i>   |

\* References: (a) J Smit and H P. J Smit, *Ferrites* (New York: John Wiley & Sons, 1959), chap. xiv; (b) J K. Galt, *Proc Inst Elec Engr.* (London), **104B**, Suppl No 5, 189 (1956); (c) K Kamigaki, *J. Phys Soc. Japan*, **16**, 1170 (1961).

grain boundaries, pores, and other magnetic imperfections. If the applied field is sinusoidal, the equation of motion leads to a loss component of magnetic susceptibility which may be written

$$\chi''_{DW}(\omega) = \chi'_{DW}(0) \left[ \frac{(\omega\beta/\alpha)}{(1 - m\omega^2/\alpha)^2 + (\omega\beta/\alpha)^2} \right], \quad (34)$$

where  $\chi'_{DW}(0)$  is the domain-wall contribution to the real part of the initial susceptibility at zero frequency.

In the limit of  $(\alpha m/\beta^2) \gg 1$ , equation (34) predicts a resonance peak in  $\chi''(\omega)$  near the angular frequency  $(\alpha/m)^2$ . However, in the limit of  $(\alpha m/\beta^2) \ll 1$ , a relaxation behavior is expected with a peak in  $\chi''(\omega)$  near the angular frequency  $\alpha/\beta$ . As we shall see later,  $\beta$  is proportional to  $[1 + (\omega\tau)^2]^{-1}$  for the most likely damping mechanisms in materials containing both divalent and trivalent iron ions. With such a frequency dependence for  $\beta$ , equation (34) predicts, under suitable circumstances, both a low-frequency relaxation and a higher-frequency resonance in agreement with the type of behavior exhibited in Figure 3, *c*. In general, the frequency of the relaxation peak in most materials seems to follow an Arrhenius relationship, i.e.,  $\alpha/\beta = (\alpha/\beta)_\infty \exp(-\Delta E/kT)$ . Table 3 gives some typical values of  $(\alpha/\beta)_\infty$  and  $\Delta E$ . At the temperatures and rotational frequencies of interstellar particles, the relaxation mechanism is probably negligible, and we shall concentrate on estimates of the effectiveness of the resonance mechanism.

It is difficult to give very reliable theoretical values for the parameters  $m$ ,  $\beta$ , and  $a$  for an arbitrary substance, but we may again appeal to experimental data for a range of estimates for  $K$ . In the limit of small frequencies, equation (34) may be written

$$\lim_{\omega \rightarrow 0} \chi''_{DW}(\omega) = \omega K_{DW} = \omega \left\{ \frac{[\chi'_{DW}(0)]^2}{\omega_{\max} \chi''_{DW}(\omega_{\max})} \right\}, \tag{35}$$

where  $\chi''_{DW}(\omega_{\max})$  is the value of  $\chi''$  at the resonance frequency  $\omega_{\max}$ . By using experimental values of the three parameters in equation (35), we may then obtain values for  $K_{DW}$ . A compilation of data is given in Table 4 for a rather diverse group of materials. The  $K_{DW}$  values range between  $10^{-11}$  and  $10^{-5}$  (rad/sec) $^{-1}$ . Unfortunately, all of these

TABLE 4

ROOM-TEMPERATURE SUSCEPTIBILITIES DUE TO DOMAIN-WALL MOTION-RESONANCE CASES

| Material  | $[\mu'(0) - 1]_{DW}$ | $[\mu''(\nu_{\max})]_{DW}$ | $\nu_{\max}$<br>(Mc/s) | $M_s$<br>(cgs) | $K \times 10^{10}$<br>(rad/sec) $^{-1}$ | Refer-<br>ence* |
|---|----------------------|----------------------------|------------------------|----------------|---|-----------------|
| NiMn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> (95 4%) <sup>†</sup>                  | 18 5                 | 13                         | 100                    | 245            | 33                                      | <i>a</i>        |
| NiMn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> (92 3%)                               | 11 4                 | 11                         | 170                    | 237            | 8 7                                     | <i>a</i>        |
| NiMn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> (90 2%)                               | 9 2                  | 10                         | 330                    | 232            | 3 3                                     | <i>a</i>        |
| NiMn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> (85 3%)                               | 6 0                  | 7 5                        | 350                    | 220            | 1 8                                     | <i>a</i>        |
| NiMn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> (70 8%)                               | 1 3                  | 3 5                        | 650                    | 183            | 0 088                                   | <i>a</i>        |
| Ni <sub>0.99</sub> Co <sub>0.01</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub>   | 3 3                  | 17                         | 270                    | 261            | 0 30                                    | <i>a</i>        |
| Ni <sub>0.964</sub> Co <sub>0.036</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> | 3 0                  | 8 5                        | 500                    | 271            | 0 26                                    | <i>a</i>        |
| Ni <sub>0.932</sub> Co <sub>0.068</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub> | 4 5                  | 4 8                        | 550                    | 277            | 0 98                                    | <i>a</i>        |
| Ni <sub>0.9</sub> Zn <sub>0.1</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub>     | 29                   | 25                         | 44                     | 392            | 97                                      | <i>a</i>        |
| Ni <sub>0.8</sub> Zn <sub>0.2</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub>     | 47                   | 48                         | 39                     | 389            | 150                                     | <i>a</i>        |
| Ni <sub>0.4</sub> Zn <sub>0.6</sub> Mn <sub>0.02</sub> Fe <sub>1.9</sub> O <sub>4</sub>     | ~800                 | ~300                       | 7                      | 323            | 39000                                   | <i>a</i>        |
| NiMn <sub>0.02</sub> Al <sub>0.98</sub> Fe <sub>1.98</sub> O <sub>4</sub>                   | ~ 0 8                | ~ 0 6                      | 800                    | 37             | 0 16                                    | <i>a</i>        |
| NiMn <sub>0.02</sub> Al <sub>1.2</sub> Fe <sub>0.8</sub> O <sub>4</sub>                     | ~ 0 6                | ~ 3                        | 250                    | 27             | 0 060                                   | <i>a</i>        |
| 3(Y <sub>2</sub> O <sub>3</sub> ) · 5(Fe <sub>2</sub> O <sub>3</sub> )                      | ~ 80                 | ~ 60                       | 12                     | 138            | 1100                                    | <i>a</i>        |
| (MgO) <sub>0.97</sub> (FeO) <sub>0.03</sub> Fe <sub>2</sub> O <sub>3</sub>                  | 29 5                 | 15 0                       | 20                     | 40 3           | 370                                     | <i>b</i>        |
| (MgO) <sub>0.95</sub> (FeO) <sub>0.05</sub> Fe <sub>2</sub> O <sub>3</sub>                  | 15 3                 | 12 5                       | 43                     | 29 0           | 56                                      | <i>b</i>        |
| (MgO) <sub>0.81</sub> (FeO) <sub>0.19</sub> Fe <sub>2</sub> O <sub>3</sub>                  | 19 8                 | 6 5                        | 55                     | 26 4           | 140                                     | <i>b</i>        |

\* References: (a) J. E. Pippin and C. L. Hogan, Sci. Rept. No. 1, Contract No. AF 19(604)1084, AFRC-TN-59-169 (ASTIA Doc No. AD 214412), Harvard University (1959); (b) G. T. Rado, V. J. Folen, and W. H. Emerson, *Proc Inst Elec. Engr (London)*, **104B**, Suppl No 5, 198 (1956); G. T. Rado, R. W. Wright, W. H. Emerson, and A. Tersis, *Phys Rev*, **88**, 909 (1952); G. T. Rado, *Rev Mod Phys*, **25**, 81 (1953).

<sup>†</sup> Variations in density of ceramic.

data have been obtained at room temperature. Reliable data at low temperatures do not seem to exist. However, the  $K$ -values at low temperatures will probably exceed those given in Table 4 since  $K_{DW}$  varies as  $\beta/a^2$ . We have seen earlier that  $\beta/a$  strongly increases with decreasing temperature and  $a$  is only weakly temperature-dependent. It may be concluded that the domain-wall mechanism would be extremely effective in particle orientation if the interstellar dust grains are greater than critical domain size.

2. *Relaxation of single-domain particles.*—It has been well established that single-domain particles exhibit a resonance absorption peak in the microwave range of frequencies (Rado 1953; Rado *et al.* 1952, 1956). As mentioned earlier, the high-frequency peaks in Figure 3 are manifestations of such "domain rotation" resonances. The mechanism leading to this absorption peak may be viewed as a ferromagnetic resonance of the domain's total magnetization in a total internal "effective magnetic field." It is this effective field which tends to stabilize the single-domain configuration. The effective field is a means of representing the effects of the internal anisotropy forces of the dipolar, crystal field, and magnetostrictive interactions.

Let us then briefly review the phenomenological treatment of ferromagnetic resonance. The equation of motion of a magnetization,  $\mathbf{M}$ , may be written

$$\frac{d}{dt}\mathbf{M} = \gamma(\mathbf{M} \times \mathbf{H}) - \mathbf{D}, \quad (36)$$

where  $\mathbf{H}$  represents all fields acting on  $\mathbf{M}$ ,  $\gamma$  is the gyromagnetic ratio ( $1.76 \times 10^7$  rad/Oer sec for "spin-only" magnetism), and  $\mathbf{D}$  stands for some general phenomenological damping term. Following Haas and Callen (1963), the damping torque may be quite generally resolved along three mutually orthogonal axes:

$$\mathbf{D} = \lambda_1(\mathbf{M} - \mathbf{M}_S) + \lambda_2\left(\frac{1}{M_S}\right)(\mathbf{M} \times \mathbf{H}) + \lambda_3\left(\frac{1}{M_S}\right)^2[\mathbf{M} \times (\mathbf{M} \times \mathbf{H})]. \quad (37)$$

These three terms have no simple, direct connection with models of magnetic-relaxation mechanisms and the  $\lambda$ 's may be complicated functions of magnetization, applied fields, frequency, sample size, etc. However, the use of this form of damping torque allows us to characterize the susceptibility of the sample in terms of a definite set of loss parameters. Later, we attempt to relate at least one of these parameters to a definite model of ferromagnetic relaxation. Two limiting cases of  $\mathbf{D}$  are commonly employed; viz.,  $\lambda_2 = \lambda_3 = 0$ , Bloch-Bloembergen damping, and  $\lambda_1 = \lambda_2 = 0$ , Landau-Lifshitz damping. The Landau-Lifshitz limit is usually found to give the most reasonable representation of the dynamic behavior of magnetic systems. The  $\lambda_2$  term is usually neglected, since it gives merely a shift in the resonance frequency.

If it is assumed that the magnetic particle is a spheroid magnetized along its axis of symmetry ( $z$ -axis), we may obtain from equations (36) and (37) the following expression for the transverse susceptibility:

$$\chi_{\pm} = \frac{M_x \pm iM_y}{H_x \pm iH_y} = \frac{\pm i\omega_M + \lambda_3}{-i(\omega \mp \omega_0) + [\lambda_1 + (\omega_0/\omega_M)\lambda_3]}, \quad (38)$$

which has the imaginary part

$$\chi''_{\pm} = \frac{\omega\lambda_3 \pm \omega_M\lambda_1}{(\omega \mp \omega_0)^2 + [\lambda_1 + (\omega_0/\omega_M)\lambda_3]^2}. \quad (39)$$

In these expressions  $\omega_M = \gamma M_S$  and  $\omega_0 = \gamma[H_A - (N_A - N_T)M_S]$ , where  $M_S$  is the saturation magnetization,  $N_A$  and  $N_T$  are the axial and transverse demagnetization factors of the spheroid, and  $H_A$  represents the effect of any forces tending to stabilize the magnetization along the axial direction. Taking equation (39) in the limit of small frequencies, we may factor out a single-domain value for  $K$  as

$$K_{SD} = \frac{1}{\omega} \lim_{\omega \rightarrow 0} \chi''_{-} = \frac{\lambda_3 - (\omega_M/\omega)\lambda_1}{\omega_0^2 + [\lambda_1 + (\omega_0/\omega_M)\lambda_3]^2}. \quad (40)$$

In view of the unreasonable behavior of equation (40) in the Bloch-Bloembergen limit, we use this expression for  $K_{SD}$  only in the Landau-Lifshitz limit, i.e.,

$$K_{SD} = \left(\frac{1}{\omega}\right) \lim_{\omega \rightarrow 0} \chi''_{-} = \frac{\lambda_3(\omega)}{\omega_0^2 + (\omega_0/\omega_M)^2\lambda_3^2}. \quad (41)$$

In general,  $\lambda_3$  is highly frequency- and temperature-dependent. To further interpret equation (41), we must turn to specific models of magnetic damping.

Equation (38) describes the dynamic response of the magnetization to small, spatially uniform transverse fields. The magnetization executes a spatially uniform precessional motion about the axial direction. The damping terms represent the net effect of all



mechanisms which lead to the transfer of energy from the coherent, uniform precessional mode into a variety of coupled thermodynamic reservoirs. For a magnetic insulator, we may identify at least three important interacting thermodynamic systems. One such reservoir is, clearly, the system of crystalline lattice vibrations or phonons. A second reservoir is found in the aggregate of modes associated with spatially non-uniform excitations of the magnetization, i.e., spin waves. Impurities in the magnetic matrix constitute a third reservoir for the direct absorption of energy from the uniform mode.

Many of the mechanisms for coupling energy out of uniform mode into these reservoirs have been discussed in the excellent reviews by Sparks (1964) and by Haas and Callen (1963). These mechanisms are usually treated at frequencies near the resonance frequency of the uniform mode, which is the situation for most common experimental interest. However, for interstellar particle relaxation, we are interested in the loss at frequencies well below the uniform-mode resonance. Thus most of the available theoretical and experimental considerations are inapplicable. To set some reasonable values on  $\lambda_3(\omega)$  in equation (41), we have considered two important mechanisms in detail: the inhomogeneous scattering of the uniform mode by fluctuations in the magnetic properties of the particle and the so-called slow relaxation (Sparks 1964) of the uniform mode by ferrous ion impurities. These two mechanisms are known to dominate the relaxation behavior of all but the most carefully prepared magnetic samples.

The inhomogeneous damping mechanism, first treated by Clogston (1958), involves the scattering of energy into spin-wave or, perhaps, even phonon modes, which may be degenerate with the frequency  $\omega$  of the uniform mode. This mechanism is essentially temperature-independent. The scattering results from random fluctuations in magnetic interactions such as dipole-dipole, magnetoelastic, and magnetocrystalline anisotropy. Undoubtedly, any magnetic grain formed in interstellar space by random processes would have large fluctuations in magnetic properties which would lead to strong inhomogeneous scattering or coupling terms. Nevertheless, we conclude that this mechanism is unimportant in magnetic-grain relaxation, since it is unlikely that there would be a large density of spin-wave states degenerate with the small frequencies of the particle rotation. Indeed, the lower bound of the major portion of the spin-wave spectrum lies, roughly, at the uniform-mode resonance frequency,  $\omega_0$ . Of course, higher-order multiple spin-wave or phonon relaxations are possible which do not require the degeneracy of  $\omega$  with the excitation spectrum. However, these multiple processes are of little importance at low temperatures and would be completely swamped by the previously mentioned ferrous ion mechanism.

Galt (see Yager, Galt, and Merritt 1955; Galt 1954, 1955, 1957; Galt and Spencer 1962) first discussed the relaxation behavior caused by ferrous ion impurities in connection with the resonance measurements on nickel ferrite doped with small amounts of ferrous ion. His original model involves the dynamic response characteristics of local  $\text{Fe}^{2+}$ - $\text{Fe}^{3+}$  complexes as the total magnetization of the substance precesses at a frequency,  $\omega$ . For  $\omega < 1/\tau$ , where  $1/\tau$  defines a "jump rate" for electronic transfer between the two ions, the system is always at equilibrium and the complex is always found in its lowest energy state. For  $\omega > 1/\tau$ , the behavior of the complex is unable to follow the motion of the magnetization. The maximum energy is transferred from the uniform mode to the complex when  $\omega\tau = 1$ . Galt (1954) derived a rather general expression for the Landau-Lifshitz damping parameter associated with such a process, which may be paraphrased in the following form:

$$\lambda_3(\omega) = cF(T) \frac{\tau}{1 + (\omega\tau)^2}. \quad (42)$$

In this expression,  $c$  is the concentration of ferrous ions present and  $F(T)$  is a complicated function of temperature which depends on angular derivatives of the contributions to the system's free energy arising from the presence of the impurities.

Recently, it has been amply demonstrated that equation (42) applies to a broad class of impurity relaxation mechanisms where  $\tau$  need only be interpreted as the time for the impurity ion to come to equilibrium (see, e.g., Spencer, LeCraw, and Linares 1961; Van Vleck 1964; Heeger, Blocker, and Ghosh 1964; Sparks 1964). These mechanisms are now usually characterized as "slow relaxations" to convey the sense that the energy exchange is only possible when we have rather slow rates of energy loss to the lattice, i.e.,  $1/\tau \approx \omega$ .

We may obtain some estimate of  $\lambda_3(\omega)$  at low temperatures from an extrapolation of ferromagnetic resonance data in this range. The full width of a resonance absorption line at half-power is given by

$$\Delta H = \frac{1}{\gamma^2} \frac{\omega_R}{M_S} \lambda_3(\omega_R) = \frac{c}{\gamma^2} \left( \frac{\omega_R}{M_S} \right) F(T) \frac{\tau}{1 + (\omega_R \tau)^2}, \tag{43}$$

where  $\omega_R$  is the measurement frequency. Between equations (42) and (43), we may eliminate  $F(T)$ , writing

$$\gamma_3(\omega) = \frac{\gamma \Delta H}{\omega_R} (\gamma M) \left[ \frac{1 + (\omega_R \tau)^2}{1 + (\omega \tau)^2} \right]. \tag{44}$$

TABLE 5

LOW-FREQUENCY LANDAU-LIFSHITZ PARAMETERS DUE TO FERROUS IONS FOR A NORMALIZED CONCENTRATION RATIO  $\text{Fe}^{2+}/\text{Fe}^{3+} = 1/10$

| Substance  | $T(^{\circ} \text{K})$ | $M_s(\text{cgs})$ | $\omega_R(\text{rad/sec})$ | $\Delta H(\text{Oer})$ | $(\gamma \Delta H)(\omega_M/\omega_R)$<br>(rad/sec) | $\tau(\text{sec})$         | $\lambda_3(\text{rad/sec})$ | References* |
|--|------------------------|-------------------|----------------------------|------------------------|---|----------------------------|-----------------------------|-------------|
| $\text{NiFe}_2\text{O}_4$                                      | 10                     | 340               | $5.77 \times 10^{10}$      | 60                     | $12 \times 10^7$                                    | $\sim 3.6 \times 10^{-11}$ | $\sim 7 \times 10^8$        | a           |
| $\text{NiFe}_2\text{O}_4$ . . . . .                            | 10                     | 340               | $15.1 \times 10^{10}$      | 20                     | $1.4 \times 10^7$                                   | $\sim 3.6 \times 10^{-11}$ | $\sim 4 \times 10^8$        | a           |
| $\text{NiFe}_2\text{O}_4$ . . . . .                            | 20                     | 340               | $5.77 \times 10^{10}$      | 90                     | $17 \times 10^7$                                    | .....                      | $> 2 \times 10^8$           | a           |
| $\text{NiFe}_2\text{O}_4$ . . . . .                            | 20                     | 340               | $15.1 \times 10^{10}$      | 40                     | $2.8 \times 10^7$                                   | .....                      | $> 3 \times 10^8$           | a           |
| $\text{MnFe}_2\text{O}_4$ . . . . .                            | 10                     | 394               | $\sim 6 \times 10^{10}$    | 340                    | $\sim 69 \times 10^7$                               | $\sim 1.7 \times 10^{-11}$ | $\sim 14 \times 10^8$       | b           |
| $\text{MnFe}_2\text{O}_4$ . . . . .                            | 20                     | 394               | $\sim 6 \times 10^{10}$    | 340                    | $\sim 69 \times 10^7$                               | $\sim 7.9 \times 10^{-12}$ | $\sim 7 \times 10^8$        | b           |
| $3\text{Y}_2\text{O}_3 \cdot 5\text{Fe}_2\text{O}_3$ . . . . . | 10                     | 170               | $5.2 \times 10^{10}$       | $\sim 10$              | $\sim 10 \times 10^7$                               | $\sim 1.2 \times 10^{-9}$  | $\gg 10^8$                  | c           |

\* References: (a) J. K. Galt and E. G. Spencer, *Phys Rev.*, **127**, 1572 (1962); (b) A. J. Heeger, T. G. Blocker III, and S. K. Ghosh, *Phys Rev.*, **134**, A399 (1964); (c) E. G. Spencer, R. C. LeCraw, and R. C. Linares, Jr., *Phys Rev.*, **123**, 1937 (1961); E. G. Spencer and R. J. Denton, *Proc Nottingham Conf on Magnetism*, p. 425 (1964).

Thus,  $\lambda_3(\omega)$  at a given temperature may be estimated from resonance measurements at that temperature. Table 5 presents a collection of relevant data at 10° and 20° K for three different magnetic oxides. All the entries in the table have been scaled to a  $\text{Fe}^{2+}:\text{Fe}^{3+}$  concentration ratio of 1:10. From this data we may safely estimate a value of  $\lambda_3$  between  $10^8$  and  $10^9$  rad/sec in the 10°–20° K temperature range for a 10 per cent ferrous ion concentration. At lower temperatures the  $\lambda_3$  values associated with the slow relaxation mechanism decrease rapidly.

If the conjectured ferromagnetic interstellar grains evolved by collision heating processes such as those described by Spitzer and Tukey (1951), ferrous ion concentrations of as high as 10 per cent or even higher would not seem unreasonable. In fact, magnetite,  $\text{Fe}_3\text{O}_4$  is probably one of the most likely stable constituents of the grains. The ferrous impurity mechanism would undoubtedly dominate the relaxation rate of single-domain particles. We may again turn to initial permeability spectra for estimates of reasonable estimates to use for  $\omega_0$  in equation (41). For those compounds which have exhibited domain rotation resonances, a range of  $\omega_0$  values between  $5 \times 10^7$  and  $2 \times 10^{10}$  rad/sec covers most, if not all, of the observations. We are led then to the single-domain estimate

$$4 \times 10^{-7} \text{ sec/rad} > K_{SD} > 2 \times 10^{-13}, \tag{45}$$

with the lower limit extremely unlikely. This estimate is several orders of magnitude greater than the ferrite value quoted by Cugnon, i.e.,  $K \simeq 10^{-14}$  sec/rad.

d) “*Super-paramagnetic*” Relaxation

A simple modification of the Davis-Greenstein model may lead to greatly enhanced  $K$ -values. As discussed earlier, DG assumed a diffuse distribution of paramagnetic iron ions throughout the dust particle. However, it would seem equally likely that the available iron atoms would aggregate themselves into many tiny magnetically ordered lumps of, say,  $\text{Fe}_3\text{O}_4$  or  $\gamma\text{-Fe}_2\text{O}_3$ . Following the arguments of Spitzer and Tukey (1951) for complex grain formation, the lumps of refractory magnetic material might be formed in the cycles of impact heating. Thus, we suggest that some of the dust particles might be viewed as a sort of “raisin pudding” of ferromagnetic clusters in a non-magnetic matrix of ice. Such clusters would be smaller than critical domain size and would behave as single-domain particles.

Fortunately, there is a good deal of experimental information which is relevant to this model. Néel (1949) first discussed the physics of this situation in connection with some problems in terrestrial magnetism. Since that time, the properties of fine-particle magnetic precipitates have been of central interest in the technology of permanent magnets. It has been found that fine single-domain particles, in the diameter range of 20–200 Å in small concentrations where interparticle interactions are unimportant exhibit so called “super-paramagnetic” behavior (see Bean and Livingston 1959; Jacobs and Bean 1963). The classification “super-paramagnetic” signifies that, at thermal equilibrium, the average magnetic moment of an assembly of such lumps follows a Langevin function of argument  $\mu H/kT$ , where  $\mu$  is the total magnetic moment of each lump and  $H$  is the applied magnetic field. Thus, at thermal equilibrium a super-paramagnetic system has an initial susceptibility per unit volume of

$$\chi_{SP} = n \left( \frac{\mu^2}{3kT} \right) = \mathfrak{N} \left( \frac{N\mu_0^2}{3kT} \right) = \mathfrak{N}\chi_0, \quad (46)$$

where the various symbols have the following meanings:

- $n$  = average number of magnetic precipitates per unit volume
- $N$  = average number of iron atoms per unit volume
- $\mathfrak{N}$  = average number of iron atoms per precipitate particle
- $\mu_0$  = “effective” magnetic moment of each magnetic ion
- $\chi_0$  = ordinary paramagnetic susceptibility for a magnetic ion concentration  $N$

The combination  $N\mu_0^2/3k$ , of course, is the Curie constant of the dust particle. We see that the susceptibility for a super-paramagnetic system is enhanced by the factor  $\mathfrak{N}$  (typically between  $10^3$  and  $10^5$ ) over that of an ordinary paramagnetic system with the same total number of magnetic atoms. If we assume a 1 atomic per cent concentration of ferric ions precipitated in ice particles,

$$\chi_{SP} = 1.7 \times 10^{-3} \left( \frac{\mathfrak{N}}{T} \right) \text{ cgs units/cm}^3. \quad (47)$$

Understanding of the orientation behavior of such particles requires some model of the dynamic super-paramagnetic response. There is rather good evidence that the time dependence of the magnetization of a super-paramagnet may be described by a rate equation

$$\frac{dM}{dt} = -\frac{1}{\tau} (M - M_{eq}), \quad (48)$$

where  $M_{\text{eq}}$  is the equilibrium value of the magnetization (i.e.,  $M_{\text{eq}} = 4\pi\chi_{SP}H$ ) and  $1/\tau$  is a relaxation rate having an Arrhenius form (Brown 1959b)

$$\frac{1}{\tau} = \nu_0 \exp(-v\Delta E/kT). \quad (49)$$

In equation (49),  $v$  is the volume of the precipitates and  $\Delta E$  is the energy barrier restraining the magnetic reversal of the individual precipitates. This barrier arises from crystal and shape anisotropies and would typically be in the range  $10^4$ – $10^6$  ergs/cm<sup>3</sup>. The factor  $\nu_0$  represents the disordering effects of random thermal modulations of magnetoelastic and magnetostatic forces which the non-magnetic matrix produces on each magnetic lump. For most substances, a value of  $\nu_0$  of the order of  $10^9$  sec<sup>-1</sup> seems quite reasonable (Bean and Livingston 1959; Jacobs and Bean 1963). In particular, Brown (1959b) has deduced the values  $(\Delta E/k) = 6.2 \times 10^{21}$  (°K/cm<sup>3</sup>) and  $\nu_0 = 5 \times 10^9$  sec<sup>-1</sup> for metallic iron precipitates. The exponential factor in equation (49) arises from the fact that the thermal field must have sufficient energy to overcome the barrier of the coercive force of each particle. For large particles, say, diameters greater than 200 Å, the magnetic moments at ordinary temperatures never surmount the barrier and the assembly exhibits permanent magnet behavior. Regardless of particle size, however, there is some temperature at which the moments become thermally "locked" leading to hysteresis effects and magnetic losses. For a harmonic time dependence of the magnetizing field we may, according to equation (48), write the imaginary component of the susceptibility as

$$\chi''_{SP}(\omega) = \chi_{SP}(0) \frac{\omega\tau}{1 + (\omega\tau)^2}. \quad (50)$$

The maximum value of  $\chi''_{SP}(\omega) = \frac{1}{2}\chi_{SP}(0)$  occurs at  $\omega\tau = 1$ . For  $\omega = 10^5$  rad/sec, we find for iron particles from equation (49) that  $\nu/T = 1.8 \times 10^3$  Å<sup>3</sup>/°K. At 10°K, a precipitate volume of  $18 \times 10^3$  Å<sup>3</sup> is required for maximum  $\chi''_{SD}$ . Using a unit-cell size of 23.5 Å<sup>3</sup> for iron and 58.9 Å<sup>3</sup> for magnetite, the corresponding  $\mathcal{N}$ -values are respectively  $1.5 \times 10^3$  and  $7.4 \times 10^2$  atoms. Using all of these values and equations (46) and (50), we obtain the following estimates of  $K$ 's for super-paramagnetic particles:

$$\begin{aligned} K_{SP} &\leq 10^{-7} \text{ sec/rad (iron precipitates)} \\ K_{SP} &\leq 10^{-6} \text{ sec/rad (magnetite precipitates)}. \end{aligned} \quad (51)$$

These  $K$  values are undoubtedly overestimates. However, the point remains that they are some six or seven orders of magnitude greater than the predictions of the DG random distribution model (eq. [32]) for the same range of iron concentrations.

#### IV. ORIENTATION BY INTERSTELLAR MAGNETIC FIELDS

We conclude with a very brief discussion of the magnetic field strengths required to orient interstellar particles by the mechanisms discussed above. In view of the many uncertainties both in the composition of the grains and in the value of magnetic dissipation, measured by  $K = \chi''/\omega$ , no very definite conclusion can be drawn, but the range of possibilities can at least be delineated.

According to recent computations by Greenberg (1966), a quantitative explanation of interstellar polarization requires that the grains be prolate spheroids of appreciable eccentricity, and moderately well oriented. The weak orientation of nearly spherical grains, assumed in the derivation of equation (22), is not applicable in this case, and no precise results are available for predicting the orientation. However, the requirement of moderate orientation evidently implies that the parameter  $\delta$  defined in the second of equations (14) be of order about unity; with this value for  $\delta$ ,  $T_{\text{av}}$  will differ appreciably

from  $T_g$ , and the effective temperatures characterizing the distribution of angular momenta about axes perpendicular and parallel, respectively, to the magnetic field,  $B$ , will differ substantially from each other. We assume in this discussion that the gas temperature  $T_g$  is appreciably different from  $T_i$ , the internal temperature of the grains.

If we assume that  $\delta$  is about unity, and let  $T_g$ ,  $n$  and  $a$  equal  $50^\circ$  K, 10 H atoms per  $\text{cm}^3$ , and  $3 \times 10^{-5}$  cm, respectively, we obtain

$$B = 5 \times 10^{-12}/K^{1/2}. \quad (52)$$

According to § IIIc, for paramagnetic relaxation  $K$  is of order  $10^{-13}$ , if we set the temperature  $T_i$  equal to  $10^\circ$  K. Thus  $B$  is in the neighborhood of  $2 \times 10^{-5}$  gauss, in general agreement with the rather high fields found originally by DG. There seems to be little prospect that the magnetic field required for orienting interstellar grains by paramagnetic relaxation can be reduced much below  $10^{-5}$  gauss.

For ferromagnetic grains the range of  $K$  is very large. The values given in Table 4 for large multiple-domain grains and in the subsequent text for single domains lie mostly in the range from  $10^{-12}$  to  $10^{-8}$ . The rough values of  $B$  obtained from equation (52) range from  $5 \times 10^{-6}$  gauss down to  $5 \times 10^{-8}$  gauss. If only the core of each grain is ferromagnetic, the values of  $B$  required for orientation would be somewhat increased, but not by orders of magnitude, if the core mass is an appreciable fraction of the total. For super-paramagnetic grains, containing clumps of about 100 iron atoms each, the computed values of  $K$  yield field strengths which can be as low as  $10^{-8}$  gauss.

We conclude that magnetic orientation of interstellar grains by magnetic fields as low as  $10^{-6}$  gauss would appear to be physically plausible, provided that iron atoms are present either in small clusters or in ferromagnetic compounds within a grain.

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