Reprinted from JOURNAL OF APPLIED PHYSICS, Vol. 34, No. 4 (Part 2), 1283–1284, April 1963

Magnetostriction Constants from Ferromagnetic Resonance*

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The saturation magnetostrictive constants of a magnetic material may be determined by means of ordinary ferromagnetic resonance. For small strains, the magnetic field required for resonance at a given frequency is a linear function of the applied uniaxial stress. The magnitude of this shift may be directly related to the magnetostrictive constants of the substance under investigation. The constants for YIG, YIG doped with Er^{3+}, YIG doped with Yb^{3+}, nickel ferrite, and nickel ferrite doped with cobalt have been measured by this new technique in a temperature range from 77° to 350°K. Where comparisons are possible, the results agree quite well with previous measurements. In particular, the measured magnetostrictive constants of YIG are in good agreement with values that have been obtained from magnetoacoustic and strain gauge experiments.

THE saturation magnetostrictive constants of a magnetic material may be determined from the strain dependence of ordinary ferromagnetic resonance. This new technique for measuring these constants has a number of distinct advantages. First, in view of the great sensitivity of ferromagnetic resonance, very small, difficult to grow crystals may be easily investigated by this method. Second, the temperature dependence of the magnetostriction may be determined over a wide temperature range. Third, and this is perhaps the most important advantage, simultaneous determinations of related phenomena such as magnetocrystalline anisotropy and magnetic relaxation may be made in a resonance experiment.

The ferromagnetic resonance frequency ω_0 may be quite generally expressed in terms of the angular derivatives of the free energy F of the substance under consideration. If the polar angles θ and ϕ represent the orientation of the magnetization, then the condition on ω_0 has the well-known form

$$\omega_0/\gamma=(M_0\sin\theta)^{-1}\left[\frac{\partial^2F}{\partial\theta^2}\frac{\partial^2F}{\partial\phi^2}-\left(\frac{\partial^2F}{\partial\theta\partial\phi}\right)^2\right]^{\frac{1}{2}},\qquad(1)$$

where M_0 is the saturation magnetization.[1,2] Also one must satisfy the auxiliary conditions that the angular first derivatives of F must vanish at the equilibrium orientation of the magnetization. In order to derive the strain dependence of ω_0 the free energy must contain a term representing the magnetoelastic interaction in addition to the usual magnetostatic and anisotropy terms.[2] For cubic materials this magnetoelastic term is usually written

$$b_1(\alpha_x^2S_{xx}+\alpha_y^2S_{yy}+\alpha_z^2S_{zz})+b_2(\alpha_x+\alpha_yS_{xy}+\alpha_y\alpha_zS_{yz}+\alpha_z\alpha_xS_{zx}),$$

where the α's are the direction cosines of the magnetization and the S's are the strain components. Al-

* Work supported by Contract AF 19(604)-5487 with the Air Force Cambridge Research Laboratories.

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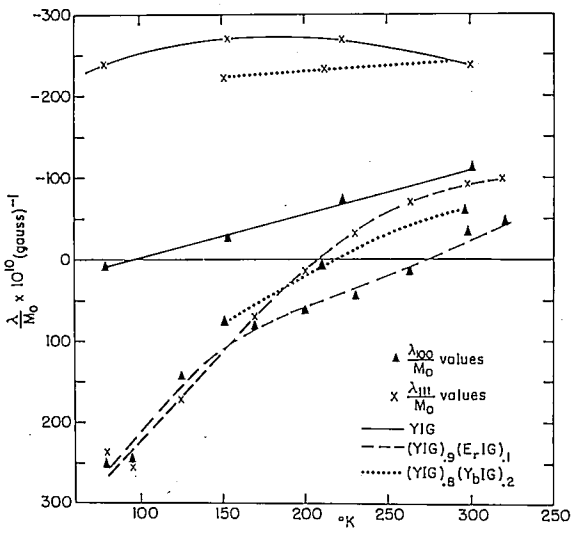

FIG. 1. Temperature dependence of the magnetostriction–magnetization ratio for YIG, YIG+10 percent Er^{3+}, and YIG+20 percent Yb^{3+}.

though the experiments are carried out at fixed stresses the derivatives in Eq. (1) have to be evaluated at constant strain since ferromagnetic resonance is ordinarily measured at frequencies much above the mechanical relaxation frequency. The stress effect is most conveniently expressed as a shift δH in the magnetic field required for resonance at a fixed measuring frequency. For small strains, this shift will be a linear function of the change in the applied stress. In general this function is quite complicated and difficult to interpret.

However for one convenient experimental configuration the magnetic field shift has a particularly simple form. If a uniaxial stress τ is applied along the $[1\bar{1}0]$ direction of a spherical sample of a cubic material, then δH for the field oriented along, respectively, the [100] and [110] direction is given as[3]

$$\delta H_{[100]} = -\left(\tfrac{3}{2}\right)(\tau/M_0)\lambda_{100},$$

$$\delta H_{[110]} = -\left(\tfrac{3}{4}\right)(\tau/M_0)(3\lambda_{111}-\lambda_{100}),$$

where the λ's are the usual magnetostriction constants. Thus, the ratios λ/M_0 may be directly determined in any insulating material where the shift at reasonable stresses is some observable fraction of the resonance linewidth—say $\delta H/\Delta H > 0.1$. For materials measured so far, this condition has not imposed any limitation on the application of the method. In general one would expect a nonuniform strain distribution over a stressed spherical sample. However, the error associated with such nonuniform distribution can be estimated since any appreciable strain inhomogeneties would give rise to a stress-dependent broadening of the resonance line.

[3] The expression for $\delta H_{[110]}$ is strictly correct only in the limit of small anisotropies.

TABLE I. Room temperature values of λ/M_0.

Material	λ_{100}/M_0(gauss)$^{-1}$	λ_{111}/M_0(gauss)$^{-1}$
$NiFe_2O_4$(296°K)	-1.62×10^{-7}	-0.72×10^{-7}
$Ni_{0.9}Co_{0.1}Fe_2O_4$(294°K)	-2.49×10^{-7}	-0.594×10^{-7}
YIG (300°K)	-1.08×10^{-8}	-2.29×10^{-8}
$(YIG)_{0.9}(ErIG)_{0.1}$(298°K)	-0.317×10^{-8}	-0.911×10^{-8}
$(YIG)_{0.8}(YbIG)_{0.2}$(296°K)	-0.60×10^{-8}	-2.40×10^{-8}

Measurements on narrow linewidth YIG indicate that the error introduced by nonuniform strain distributions is probably small.

The saturation magnetostriction constants of several magnetic oxide systems have been measured over a temperature range from 77° to 350°K by means of this resonance technique. Figure 1 and Table I present some of these data. An X-band reflection spectrometer has been used for these measurements. The stress was transmitted to the spherical sample by a quartz plunger. The observed shifts δH varied linearly with the applied stress.

The constants for YIG at room temperature—i.e., $\lambda_{100}=-1.44\times10^{-6}$ and $\lambda_{111}=-3.04\times10^{-6}$—are in substantial agreement with values from strain gauge measurements.[4] From the elastic constants of YIG,[5] the magnetoelastic constants can be obtained as $b_1=+3.48\times10^6$ and $b_2=+6.96\times10^6$ ergs/cm^3. The value of b_2 is in good agreement with the room temperature acoustical rotation value[6] of 7.4×10^6 ergs/cm^3.

It will be seen from the Fig. 1 that the addition of 10 percent Er^{3+} to YIG causes a radical change in the temperature dependence of the magnetostriction. This result is not surprising in view of the strong influence of rare-earth ions on the observed spin-lattice relaxation times of YIG. The magnitudes of both of these phenomena depend upon the strength of the strain dependence of the local crystalline field interactions. A 20 percent addition of Yb^{3+} has a relatively weaker effect on the magnetostriction of YIG.

The magnetostriction constants for nickel ferrite—i.e., $\lambda_{100}=-43.8\times10^{-6}$ and $\lambda_{111}=-19.5\times10^{-6}$—agree quite well with the value $\lambda_s=-26\times10^{-6}$ ($\lambda_s=2/5\lambda_{100}+3/5\lambda_{111}$) measured on polycrystalline samples.[7]

ACKNOWLEDGMENT

The authors wish to thank Mr. Frank N. Molea for preparing the samples used in the measurements reported here.

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