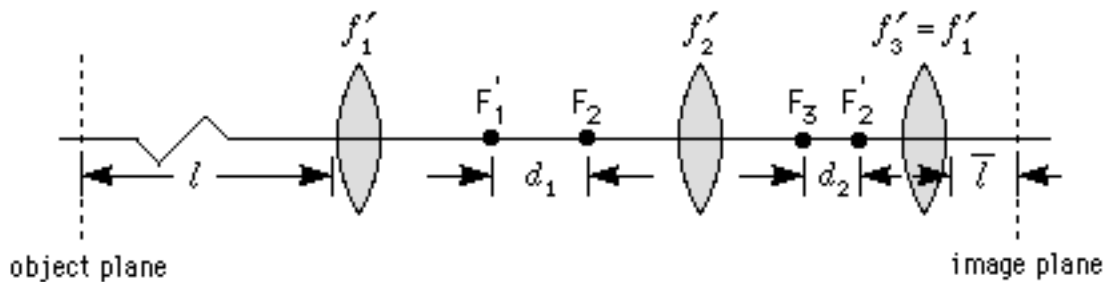


APPLIED PHYSICS 216  
OPTICAL PHYSICS AND QUANTUM ELECTRONICS  
ASSIGNMENT 1  
SPRING TERM 1999-2000

**Due Tuesday, March 14, 2000.**

1. A little "warm up" with some geometric optics and **ABCD** matrices: The image magnification or "effective focal length" of a zoom lens may be varied, in a certain range, without changing the object-to-image distance and, thus, a continuous scaling of the image may be produced. A simple, three element, reasonably good varifocal system is illustrated below.



Your task here is to analyze this system in the paraxial approximation and to examine its varifocal characteristics.

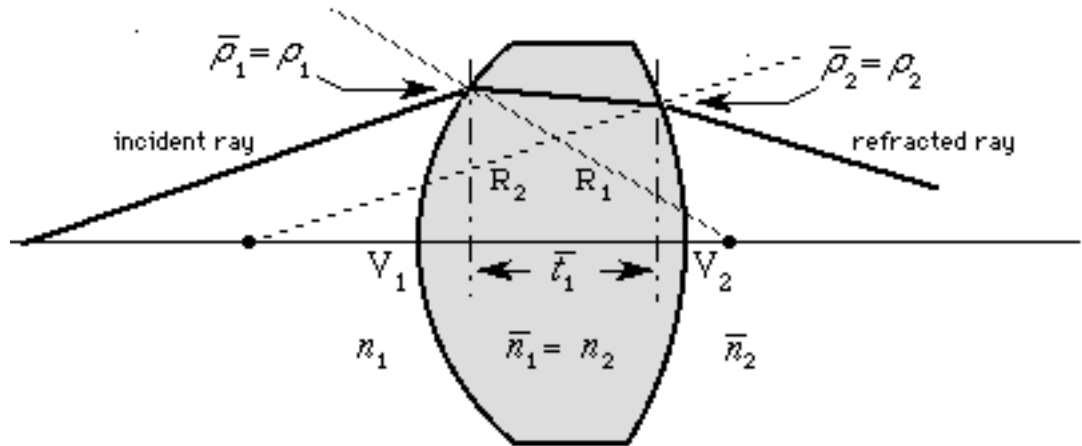
- a. Background: For brevity, let us refer to the **ABCD** matrix as a *system matrix*  $S_{lm}$  which transforms the ray from location  $m$  to location  $l$ . With this convention we may write the ray transformation equations in the form

$$\begin{pmatrix} x_2 \\ \theta_2 \end{pmatrix} = \mathbf{S}_{21} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix} = \begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} \begin{pmatrix} x_1 \\ \theta_1 \end{pmatrix}$$

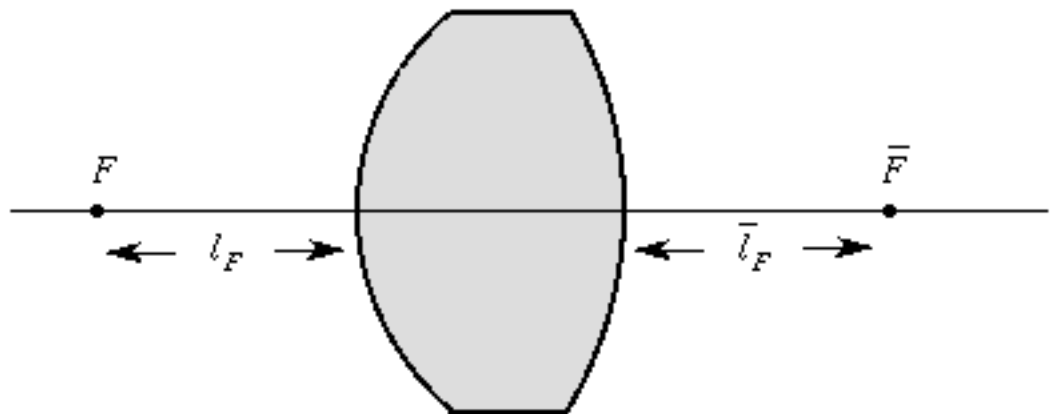
- b. More background: We illustrate below a lens formed from two spherical surfaces with radii of curvature  $R_1$  and  $R_2$ . For this lens find, in the paraxial approximation, the matrix which links ray parameters defined in the plane of the vertex  $V_1$  to those defined in the plane of the vertex  $V_2$  -- viz

$$\mathbf{S}_{V_2 V_1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

where the set of matrix elements  $\{a, b, c, d\}$  are the so called *Gaussian constants* of the lens. Find this matrix by multiplying component matrices which account for refraction at the two surfaces and the translation between surfaces. Hint: In the paraxial limit,  $\bar{l}_1$  is approximately equal to the inter-vertex distance.



c. More background: Using  $S_{V_2V_1}$  from above, show that the matrix which links ray



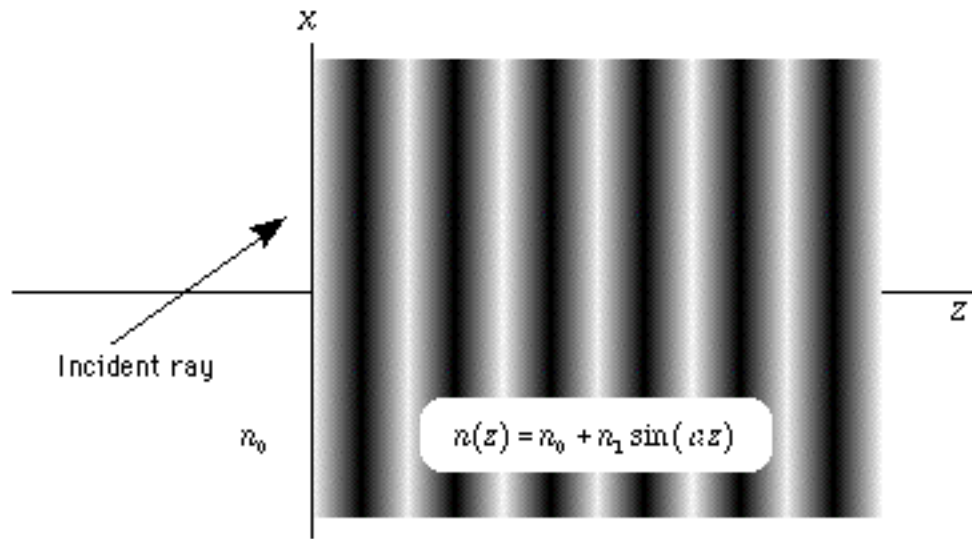
parameters defined in the front focal plane ( $F$ ) of the lens to those defined in the back focal plane ( $\bar{F}$ ) is given by

$$S_{FF} = \begin{pmatrix} 0 & \text{constant \#1} \\ \text{constant \#2} & 0 \end{pmatrix}$$

and find the constant(s).

d. Finally, **the main task**: Find the system matrix connecting points in the object and image plane for the varifocal system illustrated above. Show that image magnification can be varied by moving the middle lens while the distance between object and image planes remains relatively constant.

2. A light ray is incident upon a layered half-space as illustrated below.



- Let us suppose that the index of refraction for  $z < 0$  is constant and equal to  $n_0$ . For  $z > 0$  we suppose that it fluctuates sinusoidally and is given by  $n(z) = n_0 + n_1 \sin(\epsilon z)$  with  $n_1 < n_0$ .
- In the paraxial approximation**, find an expression for ray trajectories in the layer medium. Carefully specify the conditions under which the expression is valid.
  - Using your favorite math processor (*i.e.*, Mathematica, MATLAB, *etc.*), plot a sample set of such trajectories for the case when  $n_1 = 0.05 n_0$ .
3. We may gain considerable insight by revisiting Problem 2 as a coupled wave problem. In particular, treat the reflected wave as a field which is coupled to an incident plane wave by the spatial varying dielectric constant  $n(z) = n_0 + n_1 \sin(\epsilon z)$ . Suppose that driving incident field propagates at an angle with respect to the  $z$ -axis.
- Following the development in the lecture set *Nonlinear Optics -- Classical Picture*, generate a set of coupled Maxwell equations for the growing reflected wave and the diminishing driving field.
  - Find the condition(s) for “phase matching.” Discuss.
  - Solve the coupled Maxwell equations for the phase matched condition(s).
4. In optics a common practical problem is the focusing of the output of a laser beam onto the end face of an optical fiber located at a distance  $L$  from the laser. Assume that the

laser beam is of fundamental gaussian beam form with a beam waist (of radius  $w_1$ ) at the output of the laser. For most effective coupling, the beam should be focused at the face of the fiber to a waist of radius  $w_2$  which is smaller than the radius of the fiber. Find the distance  $d$  from the laser at which a thin lens (of focal length  $f$ ) should be located to achieve the requisite transformation. (A hint which should be taken seriously: work forward from the laser and backward from the fiber end until the gaussian beams intersect.)

5. Suppose that a Gaussian beam is propagating along a **quadratic** GRIN rod.

- a. If the beam has a **q-value** of  $q(0)$  at some particular position  $z = 0$ , show that the q-value at some position is given by<sup>†</sup>

$$\frac{1}{q(z)} = \frac{\frac{1}{q(0)} - \tan^2 z}{1 + \frac{\tan^2 z}{q(0)}}.$$

- b. Using your favorite math processor, plot the the beam curvature and width as function of position.
- c. Find an expression for dispersion relationship of the GRIN-guided Gaussian beam -- *i.e.* an expression for the longitudinal phase constant,  $\beta$ , as a function of frequency,  $\omega$ .
- d. In turn, find an expression for the group velocity of the GRIN-guided Gaussian beam -- *i.e.*  $v_g = [d\beta/d\omega]^{-1}$ .
6. The frequency separation of the modes of a cavity is very important issue in laser physics. The following exercises is a attempt to shed light on that issue.
- a. Following the discussion of asymmetric spherical (mirror) resonators in the lecture notes, show that the resonance frequencies may be written

$$\omega_{(n+m)} = \frac{c}{2L} \left( q + (1+n+m) \frac{\arccos \sqrt{u_1 u_2}}{\pi} \right)$$

<sup>†</sup> Electrical engineers in the crowd will recognize that this is the formula for wave impedance transformation in transmission line theory and, thus, *Smith Charts* may be used to find the values of  $1/q(z)$ . **If you don't understand this comment, just forget about it!**

- b. For a symmetric spherical resonator, use your favorite math processor's graphics to prepare a plot of how the following frequency differences vary with the parameter  $u$  over the range of stable values -- *i.e.*, from 0 to 2.

$$\nu_{+1}(0) - \nu(0) \quad \nu(1) - \nu(0) \quad \nu(2) - \nu(0)$$

7. In the lecture set *Descriptions of Polarized Light* we defined the Stokes vector representation of the polarization state of a light beam. As we asserted there, perhaps the most valuable attribute of this representation is its use in connection with Mueller matrices. The general idea is that effect of a given device or process on the polarization state of can be treated as a vector transformation of the form (“Mueller calculus”)

$$\begin{bmatrix} I^* \\ M^* \\ C^* \\ S^* \end{bmatrix} = [m] \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} \quad \begin{matrix} I^* & m_{11} & m_{12} & m_{13} & m_{14} & I \\ M^* & m_{21} & m_{22} & m_{23} & m_{24} & M \\ C^* & m_{31} & m_{32} & m_{33} & m_{34} & C \\ S^* & m_{41} & m_{42} & m_{43} & m_{44} & S \end{matrix}$$

where the asterisk indicates Stokes parameters transformed by propagation through a device or physical process. Of course, the Stokes parameters are defined with respect to a particular reference frame and, in particular, a frame in which the initial beam has the simplest representation. The initially defined reference frame may not be the most useful for representing the effect of a given device or physical process. You should convince yourself that the following transformation can be used to convert the initial Stokes parameters to a new reference frame at an angle  $\theta$  with respect to the initial frame:

$$\begin{bmatrix} I' \\ M' \\ C' \\ S' \end{bmatrix} = [R] \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix} \quad \begin{matrix} I' & 1 & 0 & 0 & 0 & I \\ M' & 0 & \cos 2\theta & \sin 2\theta & 0 & M \\ C' & 0 & -\sin 2\theta & \cos 2\theta & 0 & C \\ S' & 0 & 0 & 0 & 1 & S \end{matrix}$$

(It should be noted that this transform can be used to represent the effect of “optical activity”).

Thus, the compound effect of the change of reference frame and device interaction can be represented as

$$\begin{bmatrix} I^* \\ M^* \\ C^* \\ S^* \end{bmatrix} = [m][R] \begin{bmatrix} I \\ M \\ C \\ S \end{bmatrix}$$

Of course, the true value of the Mueller calculus lies in the fact that propagation through a long series of devices or physical processes is systematical tracked through a series of matrix multiplications.

As a ramp up to the main task, you should convince yourself that the following tabulation makes sense:

Device	Mueller Matrix
<p><b><u>Partial polarizer</u></b>  <math>K_1</math> and <math>K_2</math> are the intensity transmission coefficients in two orthogonal directions.</p>	$\begin{pmatrix} (K_1 + K_2) & (K_1 - K_2) & 0 & 0 \\ \frac{1}{2} (K_1 - K_2) & (K_1 + K_2) & 0 & 0 \\ 0 & 0 & 2\sqrt{K_1 K_2} & 0 \\ 0 & 0 & 0 & 2\sqrt{K_1 K_2} \end{pmatrix}$
<p><b><u>Pure retarder</u></b> -- <i>i.e.</i>, devices which alter the phase relationship between two orthogonal components of light (<math>\delta</math> is the differential retardation produced)</p>	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 1 & 0 \\ 0 & 0 & \cos \delta & \sin \delta \\ 0 & 0 & -\sin \delta & \cos \delta \end{pmatrix}$
<p><b><u>Isotropic absorber</u></b>  <math>K</math> is the intensity transmission coefficient.</p>	$\begin{pmatrix} K & 0 & 0 & 0 \\ 0 & K & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{pmatrix}$
<p><b><u>Perfect depolarizer</u></b></p>	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

**Finally, your tasks are to:**

- a. Find the Mueller matrix for specular reflection from a planar interface between two different isotropic dielectric materials.
- b. Find the Mueller matrix for transmission through a planar interface between two different isotropic dielectric materials
- c. Find the Mueller matrix for a beam propagating at an angle  $\theta$  with respect to the optic axis of uniaxial optical material.

- d. Find the Mueller matrix for a beam propagating at an angle  $\theta$  with respect to the magnetic axis in a simple magneto-optical substance.
8. In the lecture set *Optical Pulse Propagation*, we established that the optical pulse envelope for a pulse propagation in low dispersion media is described by the following **first-order pulse dispersion differential equation**:

$$b \frac{\partial^2}{\partial z^2} U(z, t) + i \frac{\partial}{\partial t} U(z, t) = 0.$$

where  $b = \frac{1}{2} \frac{d^2 n}{dk^2}$ . For many problems a **pulse dispersion integration equation** is more useful in doing calculations.

- a. Therefore derive the following pulse dispersion integration equation (I can think of two distinct ways to establish this equation):

$$U(z, t) = \int_{-\infty}^{\infty} U(z_0, t_0) \exp \left[ i b \frac{(t - t_0)^2}{2} \right] h(t - t_0) dt$$

where  $\tau = t - t_0 - v_g^{-1}(z - z_0)$ ,  $\xi = z - z_0$ , and  $h(\tau)$  is the “impulse response” function -- *i.e.* the response for a sharp spike at  $\{z = z_0, t = t_0\}$  -- given by

$$h(\tau) = \sqrt{\frac{1}{i 4 b}} \exp \left[ i b \frac{\tau^2}{2} \right].$$

- b. In studies of pulse propagation it is useful to compare the behavior of Gaussian and super-Gaussian pulse shapes. The  $m$ th-order super-Gaussians are generalizations of the Gaussian pulse shape discussed in lecture -- *i.e.*,

$$U_1(z_0, t) = \exp \left[ -\frac{1 - iC}{2} \frac{t^2}{t} \right]$$

-- that have steeper leading and trailing edges and take the form

$$U_m(z_0, t) = \exp \left[ -\frac{1+iC}{2} \frac{t}{t_0} \right]^{2m}.$$

where in both cases,  $C$  is the initial “chirp parameter.”

Using your favorite math processor to do numerical integrations of the pulse dispersion integration equation above, plot the pulse envelope shape for an initially unchirped, third-order super-Gaussian ( $m = 3$ ) at three distance -- viz.,  $z = 0, z = L_D$ , and  $z = 2L_D$  where  $L_D = t_0^2/2b = t_0^2/\omega_0^2$ . Compare these results with analytic expression derived in lecture for an initially unchirped first-order Gaussian.