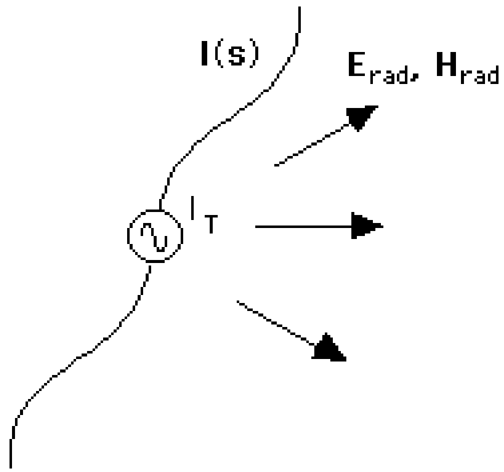
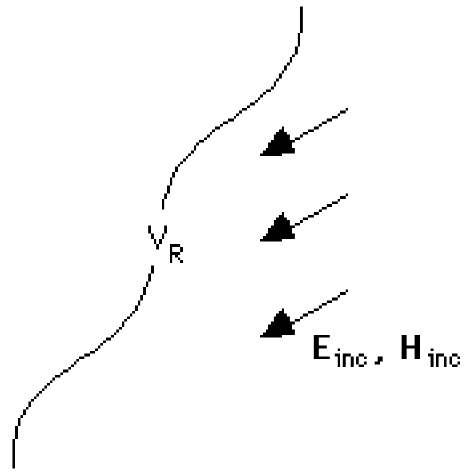
RECEIVING ANTENNA CHARACTERISTICS

To understand the characteristics and evaluate the performance of receiving antennas in general, consider the following analysis of a particular model -- *viz*. the filamentary (wire) structure illustrated below which may be used as either a **transmitting** or **receiving** antenna.



A particular antenna configuration in use as a transmitting antenna



The same antenna configuration in use as a receiving antenna

When used as a **transmitting antenna** which is excited by the indicated current generator, the current distribution $I(s)$ along the wire[1] could, in principal, be found, **self-consistently**, as a solution of Maxwell's equations which is consistent with the requirement that the component of the **electric field parallel to the wire must always be zero**! Similarly, when used as a **receiving antenna**, the **incident field excites** a *scattering field* which is the response to the requirement that the component of the electric field parallel to the wire **must be zero!** In fact, a receiving antenna may be usefully thought of as device for converting an incident field into a scattering field which may be detected in some way.

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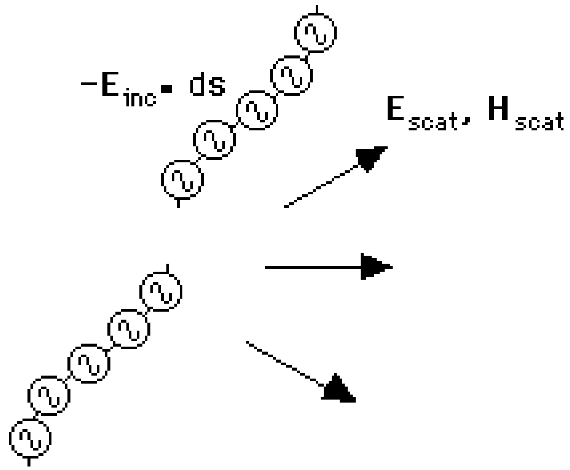
[1] Note: *s* denotes a variable position along the wire.



Re-radiated or "Scattered field" arising from currents induced on the given antenna configuration by the incident field

We now invoke the reciprocity theorem in the form

$$\iiint_{\text{all space}} \vec{\mathbf{E}}_1(\vec{\mathbf{r}}', \omega) \cdot \vec{\mathbf{J}}_2(\vec{\mathbf{r}}', \omega)\, dV' = \iiint_{\text{all space}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}', \omega) \cdot \vec{\mathbf{J}}_1(\vec{\mathbf{r}}', \omega)\, dV' \qquad [1]$$

(see Appendix for proof of the reciprocity theorem). Suppose that the index **1** refers to the field and current associated with a given antenna used as a **transmitting antenna** driven by a current source and that the index **2** refers to the field and current associated with the same antenna used as an open-circuit **receiving antenna** driven by the incident radiation -- *viz.* **the scattered field**. Consider first the following parsing of Equation [1]:

$$\iiint_{\text{all space}} \vec{\mathbf{E}}_1(\vec{\mathbf{r}}', \omega) \cdot \vec{\mathbf{J}}_2(\vec{\mathbf{r}}', \omega)\, dV' = \iiint_{\text{gap}} \vec{\mathbf{E}}_1(\vec{\mathbf{r}}', \omega) \cdot \vec{\mathbf{J}}_2(\vec{\mathbf{r}}', \omega)\, dV' + \iiint_{\text{wire}} \vec{\mathbf{E}}_1(\vec{\mathbf{r}}', \omega) \cdot \vec{\mathbf{J}}_2(\vec{\mathbf{r}}', \omega)\, dV' \qquad [2]$$

The integral **<u>over the wire is zero</u>**, because **the tangential component of E along the wire is zero** and the integral **<u>over the gap is zero</u>**, because the **output of the receiving antenna is assumed to be an open circuit** so that

$$\iiint_{\text{all space}} \vec{\mathbf{E}}_1(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_2(\vec{\mathbf{r}}',\omega)\,dV' = 0\;. \qquad [3]$$

Using this result in conjunction with the reciprocity theorem, we see that

$$\begin{aligned}&\iiint_{\text{all space}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV' \\ &\quad= \iiint_{\text{gap}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV' + \iiint_{\text{wire}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV' = 0\end{aligned} \qquad [4a]$$

or

$$\iiint_{\text{gap}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV' = -\iiint_{\text{wire}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV'\;. \qquad [4b]$$

Therefore, we may write

$$\mathrm{V_R}\;\mathrm{I_T} = -\iiint_{\text{wire}} \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV'\;. \qquad [5]$$

where $\mathrm{V_R}$ is the open-circuit voltage induced across the gap when the wire is used as a receiving antenna and $\mathrm{I_T}$ is the current source driving current on to the same wire when it is used as a transmitting antenna. Suppose that the incident field is a plane wave $\vec{\mathbf{E}}_{inc}\exp\left(-j\;\vec{\mathbf{k}}\;\vec{\mathbf{r}}\right)$ so that Equation [5] becomes

$$\mathrm{V_R} = \frac{1}{\mathrm{I_T}}\iiint_{\text{wire}} \vec{\mathbf{E}}_{inc}\;\exp\left(-j\;\vec{\mathbf{k}}\;\vec{\mathbf{r}}'\right)\;\vec{\mathbf{J}}_1(\vec{\mathbf{r}}',\omega)\,dV'\;. \qquad [6]$$

More precisely, since the incident field is perpendicular to $\vec{\mathbf{k}}$, it is the component of the current perpendicular to the propagation vector that is required -- *i.e.*

$$
\begin{aligned}
\mathrm{V_R} &= \frac{1}{\mathrm{I_T}} \iiint_{\text{wire}} \vec{\mathbf{E}}_{inc} \cdot \exp\left(-j\, \vec{\mathbf{k}} \cdot \vec{\mathbf{r}}'\right) \left\{ \vec{\mathbf{J}}_1(\vec{\mathbf{r}}', \omega) - \hat{\mathbf{k}}\left[\hat{\mathbf{k}} \cdot \vec{\mathbf{J}}_1(\vec{\mathbf{r}}', \omega)\right] \right\} dV' \\
&= -\frac{1}{\mathrm{I_T}} \iiint_{\text{wire}} \vec{\mathbf{E}}_{inc} \cdot \exp\left(-j\, \vec{\mathbf{k}} \cdot \vec{\mathbf{r}}'\right) \left\{ \hat{\mathbf{k}} \times \left[\hat{\mathbf{k}} \times \vec{\mathbf{J}}_1(\vec{\mathbf{r}}', \omega)\right] \right\} dV' \\
&= -\vec{\mathbf{E}}_{inc} \cdot \left\{ \hat{\mathbf{k}} \times \left[\hat{\mathbf{k}} \times \frac{1}{\mathrm{I_T}} \iiint_{\text{wire}} \vec{\mathbf{J}}_1(\vec{\mathbf{r}}', \omega) \exp\left(-j\, \vec{\mathbf{k}} \cdot \vec{\mathbf{r}}'\right) dV' \right] \right\}
\end{aligned}
\quad . \qquad [7]
$$

Recall our treatment of the far-fields of a transmitting antenna, the electric field strength is given by

$$
\vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = \frac{j\eta\, k}{4\pi} \frac{\exp(-j\, k\, r)}{r} \left\{ \hat{\mathbf{r}} \times \left[\hat{\mathbf{r}} \times \vec{\mathbf{N}}(\vec{\mathbf{r}}, \omega) \right] \right\} \qquad [8a]
$$

where

$$
\vec{\mathbf{N}}(\vec{\mathbf{r}}, \omega) = \iiint \exp\left[j\, k\, \hat{\mathbf{r}} \cdot \vec{\mathbf{r}}'\right] \vec{\mathbf{J}}(\vec{\mathbf{r}}', \omega)\, dV' \ . \qquad [8b]
$$

Thus, Equation [7] may be written

$$
\mathrm{V_R} = -\vec{\mathbf{E}}_{inc} \cdot \left\{ \hat{\mathbf{k}} \times \left[\hat{\mathbf{k}} \times \frac{1}{\mathrm{I_T}} \vec{\mathbf{N}}_{trans}(\omega) \right] \right\} . \qquad [7]
$$

where $\vec{\mathbf{N}}_{trans}(\omega)$ is the **current transform** of the antenna. In words the basically idea is simple: **The received open-circuit voltage is maximized when the field incident on a given antenna most resembles the radiation-zone field which that antenna would radiate as a transmitting antenna.**

APPENDIX: PROOF OF THE RECIPROCITY THEOREM

Consider the following vector identity for arbitrary vectors:

$$\vec{\ } \left[\vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega)\right] = \vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega) \quad \vec{\ } \times\vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega) - \vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega) \quad \vec{\ } \times\vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega) \qquad \text{[A-1a]}$$

$$\vec{\ } \left[\vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega)\right] = \vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega) \quad \vec{\ } \times\vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega) \quad \vec{\ } \times\vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega) \qquad \text{[A-1a]}$$

Using these two expressions in conjunction with the macroscopic Maxwell's equations for linear, isotropic media, we obtain

$$\vec{\ } \left[\vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega)\right] = \vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega) \ \left[-j\,\mu\,\omega\,\vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega)\right] - \vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega) \ \left[\vec{\mathbf{J}}_2(\vec{\mathbf{r}},\omega) + j\,\varepsilon\,\omega\,\vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega)\right] \qquad \text{[A-2a]}$$

$$\vec{\ } \left[\vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega)\right] = \vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega) \ \left[-j\,\mu\,\omega\,\vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega)\right] - \vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega) \ \left[\vec{\mathbf{J}}_1(\vec{\mathbf{r}},\omega) + j\,\varepsilon\,\omega\,\vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega)\right] \qquad \text{[A-2b]}$$

Adding these two equations, we see that

$$\vec{\ } \left[\vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_2(\vec{\mathbf{r}},\omega) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega)\times\vec{\mathbf{H}}_1(\vec{\mathbf{r}},\omega)\right] = \vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega) \ \ \vec{\mathbf{J}}_1(\vec{\mathbf{r}},\omega) - \vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega) \ \ \vec{\mathbf{J}}_2(\vec{\mathbf{r}},\omega) \qquad \text{[A-3]}$$

and then integrating the resulting equation over some arbitrary volume, we obtain

$$\begin{aligned} &\iiint \vec{\ } \left[\vec{\mathbf{E}}_1(\vec{\mathbf{r}}',\omega)\times\vec{\mathbf{H}}_2(\vec{\mathbf{r}}',\omega) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}',\omega)\times\vec{\mathbf{H}}_1(\vec{\mathbf{r}}',\omega)\right] d\,V' \\ &\qquad = \iiint \left[\vec{\mathbf{E}}_2(\vec{\mathbf{r}},\omega) \ \ \vec{\mathbf{J}}_1(\vec{\mathbf{r}},\omega) - \vec{\mathbf{E}}_1(\vec{\mathbf{r}},\omega) \ \ \vec{\mathbf{J}}_2(\vec{\mathbf{r}},\omega)\right] d\,V' \end{aligned} \qquad \text{[A-4]}$$

Further, using Gauss's Theorem on LHS of this equation yields

$$\iint \left[\vec{\mathbf{E}}_1(\vec{\mathbf{r}}', \omega) \times \vec{\mathbf{H}}_2(\vec{\mathbf{r}}', \omega) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}', \omega) \times \vec{\mathbf{H}}_1(\vec{\mathbf{r}}', \omega) \right] \, d\vec{\mathbf{S}}$$
$$= \iiint \left[\vec{\mathbf{E}}_2(\vec{\mathbf{r}}, \omega) \;\; \vec{\mathbf{J}}_1(\vec{\mathbf{r}}, \omega) - \vec{\mathbf{E}}_1(\vec{\mathbf{r}}, \omega) \;\; \vec{\mathbf{J}}_2(\vec{\mathbf{r}}, \omega) \right] d\,V' \qquad \text{[A-5]}$$

"In many instances", we may argue that the LHS of this equation vanishes -- *e.g.* in most instances the fields may be taken to be essentially plane waves at very large distances and, therefore,

$$\boxed{\iiint \vec{\mathbf{E}}_2(\vec{\mathbf{r}}, \omega) \;\; \vec{\mathbf{J}}_1(\vec{\mathbf{r}}, \omega) \, d\,V' = \iiint \vec{\mathbf{E}}_1(\vec{\mathbf{r}}, \omega) \;\; \vec{\mathbf{J}}_2(\vec{\mathbf{r}}, \omega) \, d\,V'} \qquad \text{[A-6]}$$

which is one form of the **famous reciprocity theorem**.