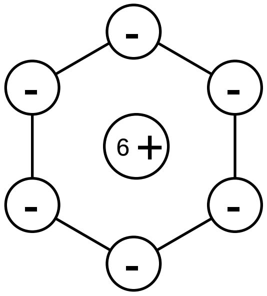
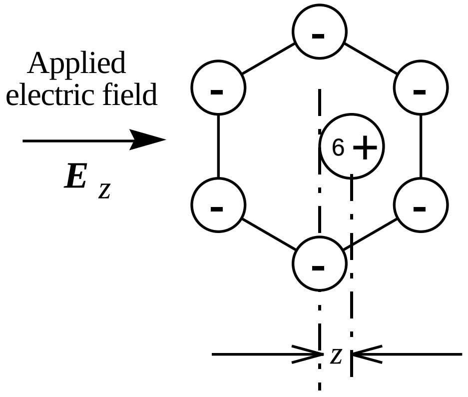
ELECTROMAGNETIC PROPERTIES OF MATTER

I. DYNAMICS OF BOUND CHARGES - MODELING DIELECTRIC BEHAVIOR:

Consider the electrically induced distortion or *polarization* of some typical **neutral** and isolated "molecular" configuration[1]-- *viz*.



EQUILIBRIUM CONFIGURATION



DISTORTED CONFIGURATION

For small distortions[2], the motion of the effective charge displacement -- *i.e.* the displacement of the respective centers of mass of the positive and negative charges -- may be described in terms of the equation of a driven, damped harmonic oscillator -- *viz*.

$$m_{eff}\,\ddot{z} = -k_{eff}\,z + q_{eff}\,\boldsymbol{E}_z(t) - d_{eff}\,\dot{z} \qquad \text{[I-1]}$$

where m_{eff} is the effective mass of the effective charge q_{eff}, k_{eff}.is the effective restoring force, and d_{eff} .is the effective damping constant.

If
$$\boldsymbol{E}_z(t) = \boldsymbol{Re}\left\{\boldsymbol{E}_z(\omega)\,\exp(j\,\omega\,t)\right\} \qquad \text{[I-2a]}$$

then
$$\left[-\omega^2 + \omega_{\text{res}}^2 + j\,\omega/\tau_{\text{damp}}\right] z(\omega) = \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_z(\omega) \qquad \text{[I-2b]}$$

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[1] It might be a make-believe carbon atom or a tungsten hexafluoride molecule.

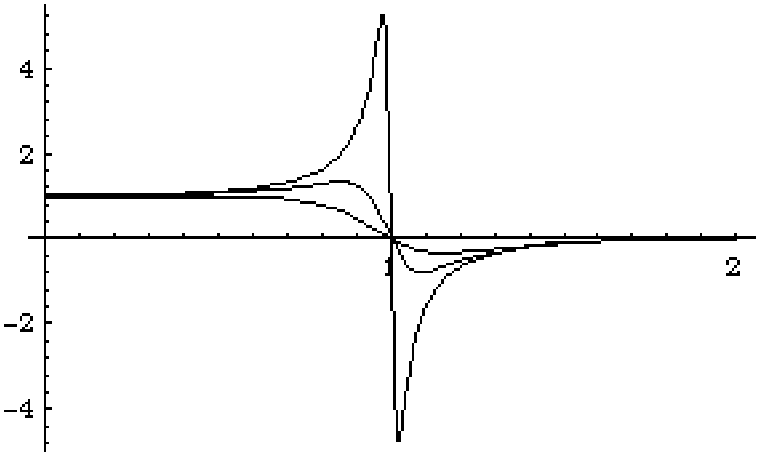
[2] That is, when the applied field is small compared to the *internal* or *bonding* field.

where $\omega_{\text{res}}^2 = k_{eff}/m_{eff}$ and $1/\tau_{\text{damp}} = d_{eff}/m_{eff}$. Thus, the frequency dependent **molecular polarizability** is given by

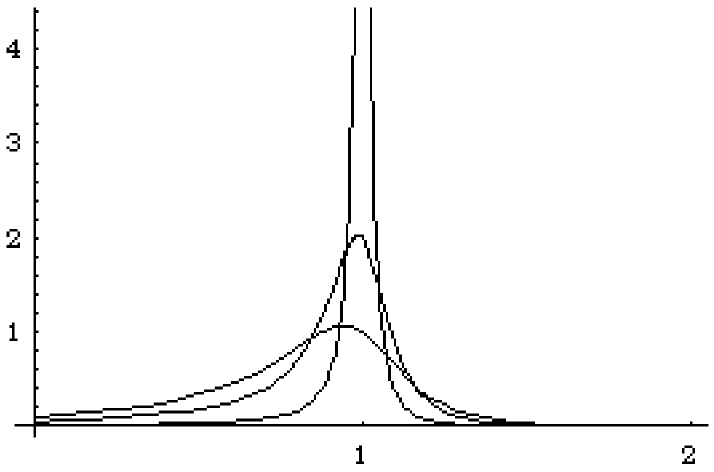
$$\begin{aligned}\alpha(\omega) &\equiv \frac{\text{induced electric dipole moment at }\omega}{\text{effective applied electric field at }\omega} = \frac{p(\omega)}{\boldsymbol{E}_z(\omega)} \\ &= \frac{q_{eff}\; z(\omega)}{\boldsymbol{E}_z(\omega)} = \left[q_{eff}^2/m_{eff}\right]\left[-\omega^2 + \omega_{\text{res}}^2 + j\,\omega/\tau_{\text{damp}}\right]^{-1} \qquad \text{[I-3]} \\ &= \alpha'(\omega) - j\,\alpha''(\omega)\end{aligned}$$

where
$$\alpha'(\omega) = \left[q_{eff}^2/m_{eff}\right]\left[-\omega^2 + \omega_{\text{res}}^2\right]\left[\left(-\omega^2 + \omega_{\text{res}}^2\right)^2 + \left(\omega/\tau_{\text{damp}}\right)^2\right]^{-1} \qquad \text{[I-4a]}$$

and
$$\alpha''(\omega) = \left[q_{eff}^2/m_{eff}\right]\left[\omega/\tau_{\text{damp}}\right]\left[\left(-\omega^2 + \omega_{\text{res}}^2\right)^2 + \left(\omega/\tau_{\text{damp}}\right)^2\right]^{-1} \qquad \text{[I-4b]}$$



$\alpha'(\omega)\left[q_{eff}^2/k_{eff}\right]^{-1}$ *vs.* $\log_{10}\left[\omega/\omega_{\text{res}}\right]$
(for $\omega_{\text{res}}\tau_{\text{damp}} = 1, 2, \text{and } 10$)



$\alpha''(\omega)\left[q_{eff}^2/k_{eff}\right]^{-1}$ *vs.* $\log_{10}\left[\omega/\omega_{\text{res}}\right]$
(for $\omega_{\text{res}}\tau_{\text{damp}} = 1, 2, \text{and } 10$)

Here these expressions are written in dimensionless form -- *viz.*

$$\alpha'(\omega) = \left[q_{eff}^2/k_{eff}\right]\left[1-\omega^2/\omega_{\text{res}}^2\right]\left[\left(1-\omega^2/\omega_{\text{res}}^2\right)^2 + \left(\omega/\omega_{\text{res}}\right)^2\left(\omega_{\text{res}}\tau_{\text{damp}}\right)^{-2}\right]^{-1} \qquad \text{[I-5a]}$$

and $\alpha''(\omega) = \left[q_{eff}^2/k_{eff}\right]\left[\left(\omega/\omega_{\text{res}}\right)\left(\omega_{\text{res}}\tau_{\text{damp}}\right)^{-1}\right]\left[\left(1-\omega^2/\omega_{\text{res}}^2\right)^2 + \left(\omega/\omega_{\text{res}}\right)^2\left(\omega_{\text{res}}\tau_{\text{damp}}\right)^{-2}\right]^{-1}$ [I-5b]

To complete the dynamic model, it is clear that the relative charge motion within each polarizable "molecule" makes a contribution $q_{eff}\ \dot{z}(t)$ or $\dot{p}(t)$ to the total current flow.[3] The aggregated current flow associated with the induced polarization of many molecules leads to a current density

$$\left[\vec{\mathbf{J}}_{\text{bound}}(t)\right]_z = \sum_i N_i\ q_i\ \dot{z}_i(t) = \sum_i N_i\ \dot{p}_i(t) \quad \Leftrightarrow \quad \left[\vec{\mathbf{J}}_{\text{bound}}(\omega)\right]_z = j\,\omega \sum_i N_i\ q_i\ z_i(\omega) = j\omega \sum_i N_i\ p_i(\omega)$$

[I-6]

where $p_i(t)$ is the induced dipole moment of the *ith* type of "molecule" and N_i is number of such molecules per unit volume. More generally, we may write the current density associated with a **polarization density** as

$$\vec{\mathbf{J}}_{\text{bound}}(\vec{\mathbf{r}}, t) = \frac{\partial}{\partial t}\vec{\mathbf{P}}(\vec{\mathbf{r}}, t) \quad \Leftrightarrow \quad \vec{\mathbf{J}}_{\text{bound}}(\vec{\mathbf{r}}, \omega) = j\,\omega\,\vec{\mathbf{P}}(\vec{\mathbf{r}}, \omega) \qquad \text{[I-7]}$$

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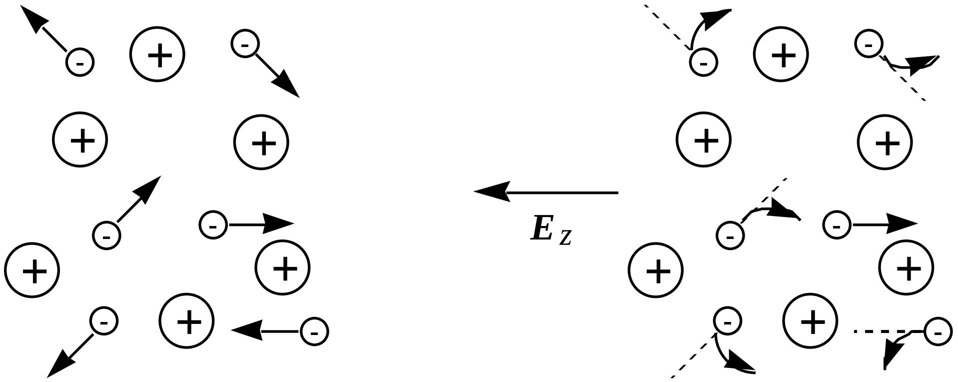
[3] In applying our model of polarizability, we have for simplicity neglected "internal field effects" -- *i.e.* changes in the applied field due dipole-dipole interactions. When included, one obtains the Clausius-Mossotti relationship -- *viz.*

$$N\,\alpha = 3\,\chi\,\varepsilon_0/\left(\chi + 3\,\varepsilon_0\right)$$

where χ is the macroscopic *dielectric susceptibility.*

II. DYNAMICS OF UNBOUND CHARGES - MODELING CONDUCTIVE BEHAVIOR:

Consider the motion induced in a swarm of charged particles by an applied electric field.



EQUILIBRIUM MOTION INDUCED "DRIFT" MOTION

For the **drift velocity** associated with some mean, effective charge carrier the following equation of motion seems plausible:[4]

$$m_{eff}\,\dot{v}_z(t) = q_{eff}\,\boldsymbol{E}_z(t) - d_{eff}\,v_z(t) \qquad \text{[II-1]}$$

If

$$\boldsymbol{E}_z(t) = \boldsymbol{Re}\left\{\boldsymbol{E}_z(\omega)\,\exp\left(j\,\omega t\right)\right\} \qquad \text{[II-2a]}$$

then

$$\left[j\,\omega + 1/\tau_{\text{coll}}\right] v_z(\omega) = \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_z(\omega) \qquad \text{[II-2b]}$$

where $1/\tau_{\text{coll}} = d_{eff}/m_{eff}$. Thus, the frequency dependent **conductivity** is given by

$$\begin{aligned}\sigma(\omega) &\equiv \frac{\text{induced current density at }\omega}{\text{applied electric field at }\omega} = \sigma'(\omega) - j\,\sigma''(\omega)\\ &= \frac{N_{eff}\;q_{eff}\;v_z(\omega)}{\boldsymbol{E}_z(\omega)} = \left[\frac{N_{eff}\;q_{eff}^2\;\tau_{\text{coll}}}{m_{eff}}\right]\left[1 + j\,\omega\,\tau_{\text{coll}}\right]^{-1}\end{aligned} \qquad \text{[II-3]}$$

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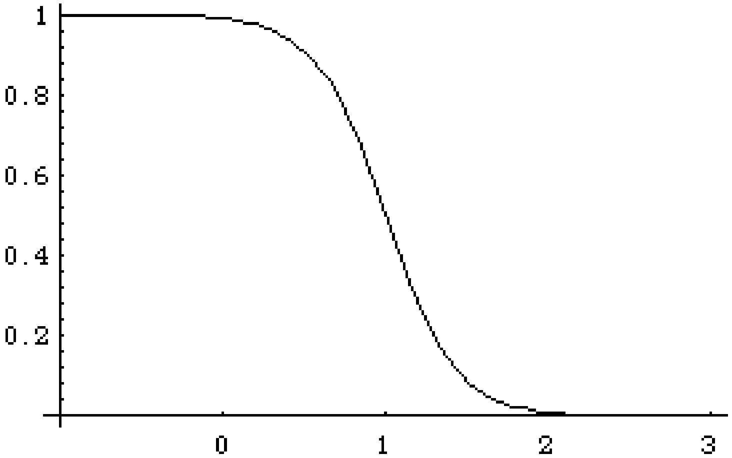
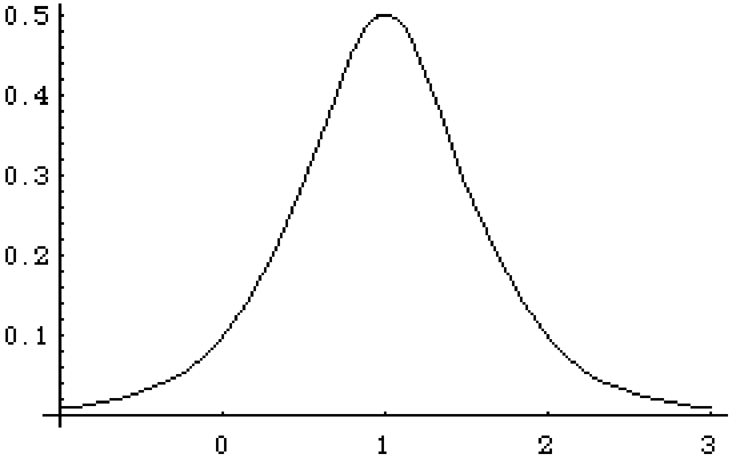
[4] Again for simplicity, the restoring force(s) associated with "plasma effects" is (are) not included in this expression.

where $$\sigma'(\omega) = \left[\frac{N_{eff}\ q_{eff}^2\ \tau_{\text{coll}}}{m_{eff}}\right]\left[1+(\omega\tau_{\text{coll}})^2\right]^{-1} \qquad \text{[II-4a]}$$

and $$\sigma'(\omega) = \left[\frac{N_{eff}\ q_{eff}^2\ \tau_{\text{coll}}}{m_{eff}}\right]\left[\omega\tau_{\text{coll}}\right]\left[1+(\omega\tau_{\text{coll}})^2\right]^{-1} \qquad \text{[II-4b]}$$



$\sigma'(\omega)\left[\frac{N_{eff}\ q_{eff}^2\ \tau_{\text{coll}}}{m_{eff}}\right]^{-1}$ *vs.* $\log_{10}\left[\omega\tau_{\text{coll}}\right]$



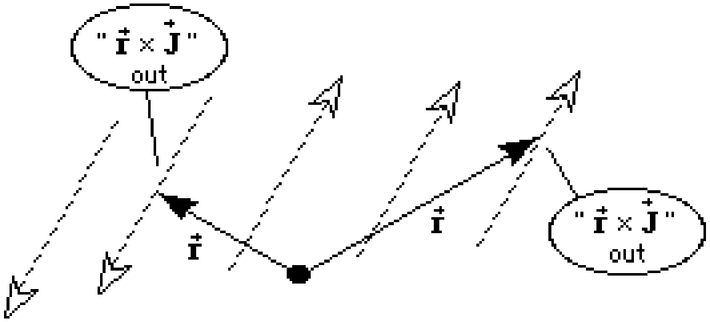
$\sigma''(\omega)\left[\frac{N_{eff}\ q_{eff}^2\ \tau_{\text{coll}}}{m_{eff}}\right]^{-1}$ *vs.* $\log_{10}\left[\omega\tau_{\text{coll}}\right]$

III. MAGNETIC PROPERTIES:

If there are microscopic current components such that the integral

$$\iiint \vec{\mathbf{r}} \times \vec{\mathbf{J}}\, dV \qquad \text{[III-1]}$$

over some region of atomic dimensions does not vanish -- *e.g.*



-- these components may be represented as

$$\vec{\mathbf{J}}_{mag}(\vec{\mathbf{r}}, t) = \vec{\nabla} \times \vec{\mathbf{M}}(\vec{\mathbf{r}}, t) \qquad \text{[III-2]}$$

where $\vec{\mathbf{M}}(\vec{\mathbf{r}}, t) \equiv \frac{1}{2} \iiint \vec{\mathbf{r}} \times \vec{\mathbf{J}}\, dV$ is called the **magnetization** or **magnetic moment density.** Thus, the macroscopic form of Ampère's law may be expressed in the form

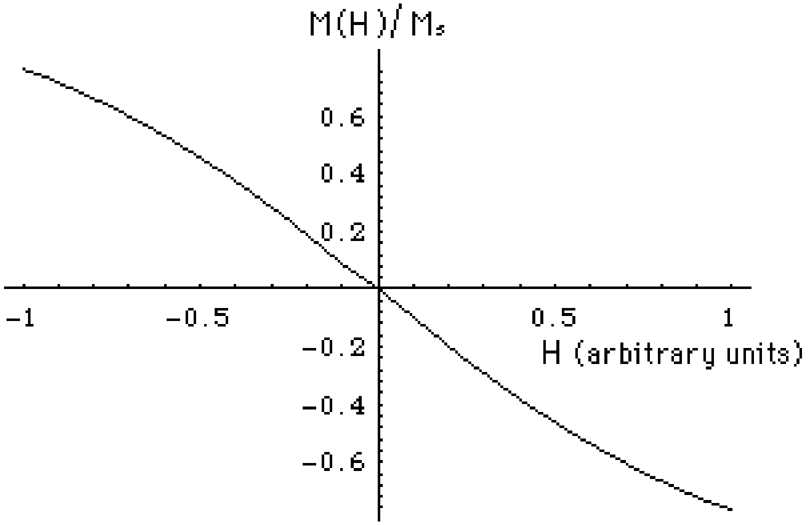
$$\vec{\nabla} \times \vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \mu_0\, \vec{\mathbf{J}}(\vec{\mathbf{r}}, t) + \mu_0\, \vec{\nabla} \times \vec{\mathbf{M}}(\vec{\mathbf{r}}, t) + \mu_0 \frac{\partial}{\partial t} \vec{\mathbf{P}}(\vec{\mathbf{r}}, t) + \mu_0\, \varepsilon_0 \frac{\partial}{\partial t} \vec{\mathbf{E}}(\vec{\mathbf{r}}, t) \qquad \text{[III-3a]}$$

or

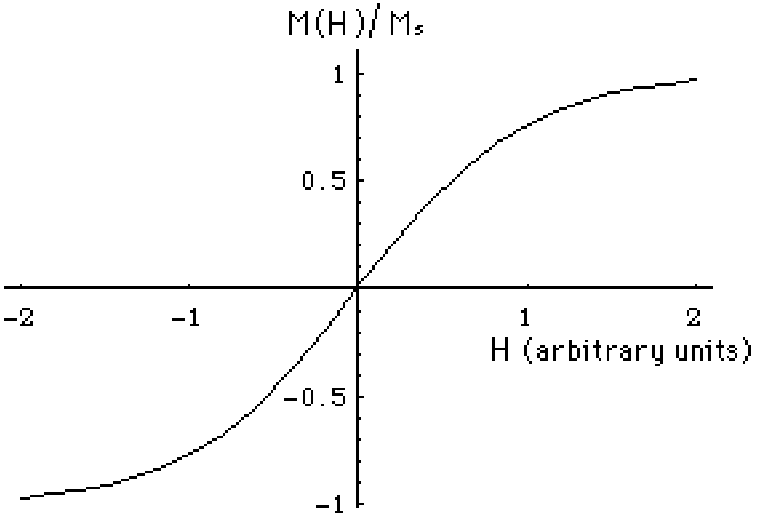
$$\vec{\nabla} \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{J}}(\vec{\mathbf{r}}, t) + \frac{\partial}{\partial t} \vec{\mathbf{D}}(\vec{\mathbf{r}}, t) \qquad \text{[III-3b]}$$

where $\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \mu_0 \left[\vec{\mathbf{H}}(\vec{\mathbf{r}}, t) + \vec{\mathbf{M}}(\vec{\mathbf{r}}, t)\right]$ and $\vec{\mathbf{E}}(\vec{\mathbf{r}}, t) = (1/\varepsilon_0)\left[\vec{\mathbf{D}}(\vec{\mathbf{r}}, t) - \vec{\mathbf{P}}(\vec{\mathbf{r}}, t)\right]$.

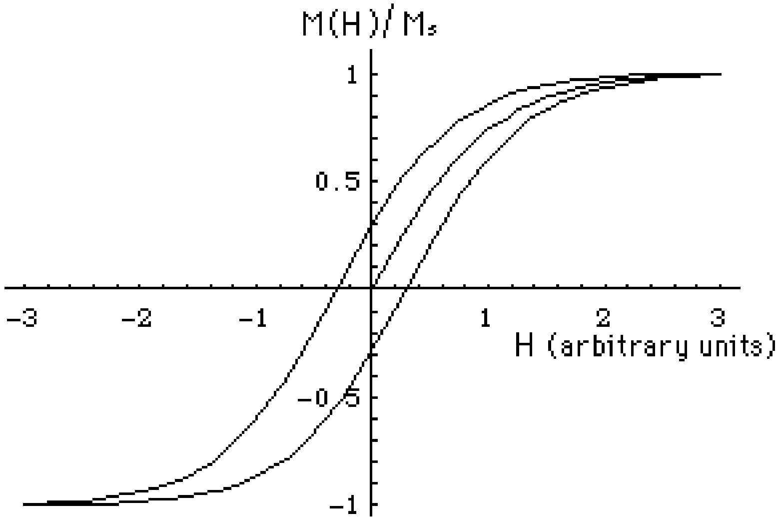
CLASSES OF MAGNETIC MATERIALS:



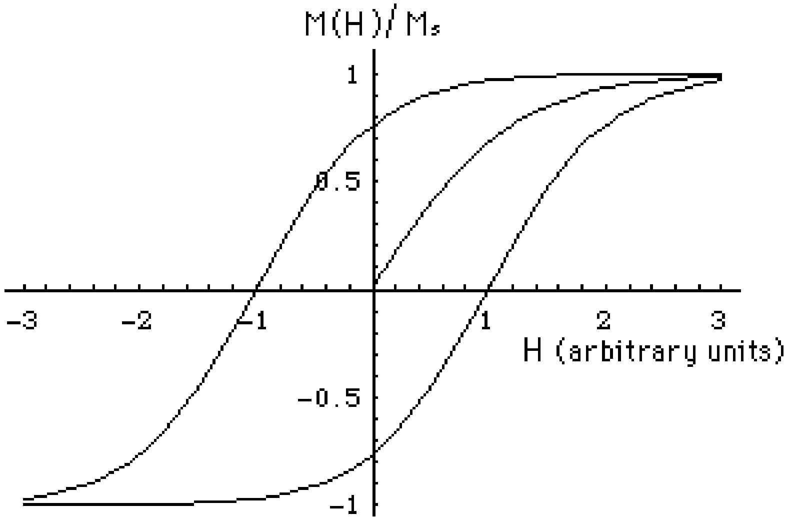
DIAMAGNETIC MATERIAL



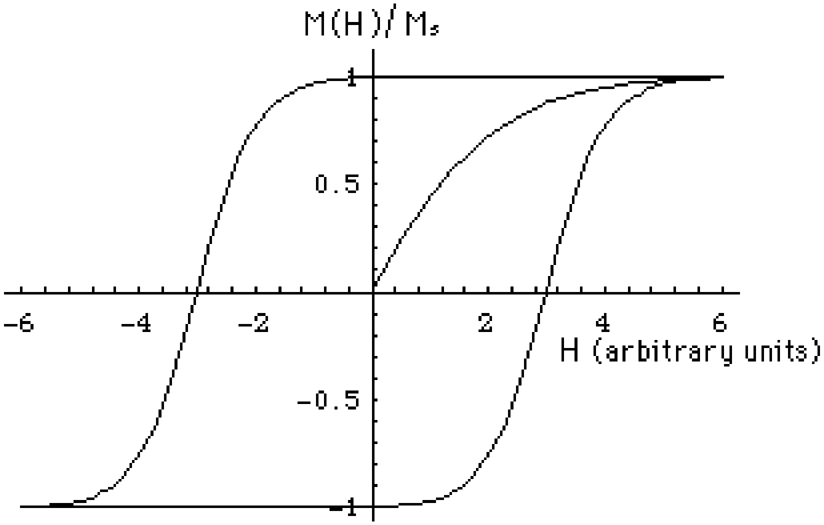
PARAMAGNETIC MATERIAL



"SOFT" FERROMAGNETIC MATERIAL



"HARD" FERROMAGNETIC MATERIAL



"SQUARE-LOOP" FERROMAGNETIC MATERIAL

IV. DYNAMICS OF MAGNETOPLASMAS:

Consider the motion induced in a swarm of charged particles in a static magnetic field, $\vec{\mathbf{B}}_c$, by a time-varying applied electric field, $\vec{\mathbf{E}}(t)$. For the **drift velocity** associated with some mean, effective charge carrier the following equation of motion seems plausible:

$$m_{eff}\ \dot{\vec{\mathbf{v}}}(t) = q_{eff}\ \vec{\mathbf{E}}(t) - d_{eff}\ \vec{\mathbf{v}}(t) + q_{eff}\ \vec{\mathbf{v}}(t) \times \vec{\mathbf{B}}_c \qquad \text{[IV-1]}$$

If

$$\vec{\mathbf{E}}(t) = \boldsymbol{Re}\left\{\vec{\mathbf{E}}(\omega)\ \exp(j\ \omega t)\right\} \qquad \text{[IV-2a]}$$

then

$$\left[j\ \omega + 1/\tau_{\text{coll}}\right]\vec{\mathbf{v}}(\omega) + \vec{\omega}_c \times \vec{\mathbf{v}}(\omega) = \left[q_{eff}/m_{eff}\right]\vec{\mathbf{E}}(\omega) \qquad \text{[IV-2b]}$$

where $\vec{\omega}_c = \left[q_{eff}/m_{eff}\right]\vec{\mathbf{B}}_c$ and $1/\tau_{\text{coll}} = d_{eff}/m_{eff}$. Resolving this vector equation into a set of component equations, we see that

$$\left[j\,\omega + 1/\tau_{\text{coll}}\right] v_x(\omega) - \omega_c\, v_y(\omega) = \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_x(\omega) \qquad \text{[IV-3a]}$$

$$\left[j\,\omega + 1/\tau_{\text{coll}}\right] v_y(\omega) + \omega_c\, v_x(\omega) = \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_y(\omega) \qquad \text{[IV-3b]}$$

$$\left[j\,\omega + 1/\tau_{\text{coll}}\right] v_z(\omega) = \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_z(\omega) \qquad \text{[IV-3c]}$$

Solving for $v_x(\omega)$ and $v_y(\omega)$

$$\left\{\left[j\,\omega + 1/\tau_{\text{coll}}\right]^2 + \omega_c^2\right\} v_x(\omega) = \left[j\,\omega + 1/\tau_{\text{coll}}\right]\left[q_{eff}/m_{eff}\right] \boldsymbol{E}_x(\omega) + \omega_c \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_y(\omega) \qquad \text{[IV-4a]}$$

$$\left\{\left[j\,\omega + 1/\tau_{\text{coll}}\right]^2 + \omega_c^2\right\} v_y(\omega) = \left[j\,\omega + 1/\tau_{\text{coll}}\right]\left[q_{eff}/m_{eff}\right] \boldsymbol{E}_y(\omega) - \omega_c \left[q_{eff}/m_{eff}\right] \boldsymbol{E}_x(\omega) \qquad \text{[IV-4b]}$$

Thus, the relationship between current density and electric field -- *i.e.* Ohm's law -- is a linear tensor relationship -- *viz*.

$$\begin{bmatrix} J_x(\omega) \\ J_y(\omega) \\ J_z(\omega) \end{bmatrix} = \begin{bmatrix} \sigma_{xx}(\omega) & \sigma_{xy}(\omega) & 0 \\ \sigma_{yx}(\omega) & \sigma_{yy}(\omega) & 0 \\ 0 & 0 & \sigma_{zz}(\omega) \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix} \qquad \text{[IV-5]}$$

and the components of the frequency dependent, **tensor conductivity** are given by

$$\sigma_{xx}(\omega) = \left[N_{eff}\ q_{eff}^2/m_{eff}\right]\left[j\,\omega + 1/\tau_{\text{coll}}\right]\left\{\left[j\,\omega + 1/\tau_{\text{coll}}\right]^2 + \omega_c^2\right\}^{-1} \qquad \text{[IV-6a]}$$

$$\sigma_{yy}(\omega) = \left[N_{eff}\ q_{eff}^2/m_{eff}\right]\left[j\,\omega + 1/\tau_{\text{coll}}\right]\left\{\left[j\,\omega + 1/\tau_{\text{coll}}\right]^2 + \omega_c^2\right\}^{-1} \qquad \text{[IV-6b]}$$

$$\sigma_{xy}(\omega) = -\sigma_{yx}(\omega) = \left[N_{eff}\ q_{eff}^2/m_{eff}\right]\left[\omega_c\right]\left\{\left[j\,\omega + 1/\tau_{\text{coll}}\right]^2 + \omega_c^2\right\}^{-1} \qquad \text{[IV-6c]}$$

$$\sigma_{zz}(\omega) = \left[N_{eff}\ q_{eff}^2/m_{eff}\right]\left[j\,\omega + 1/\tau_{\text{coll}}\right]^{-1} \qquad \text{[IV-6d]}$$