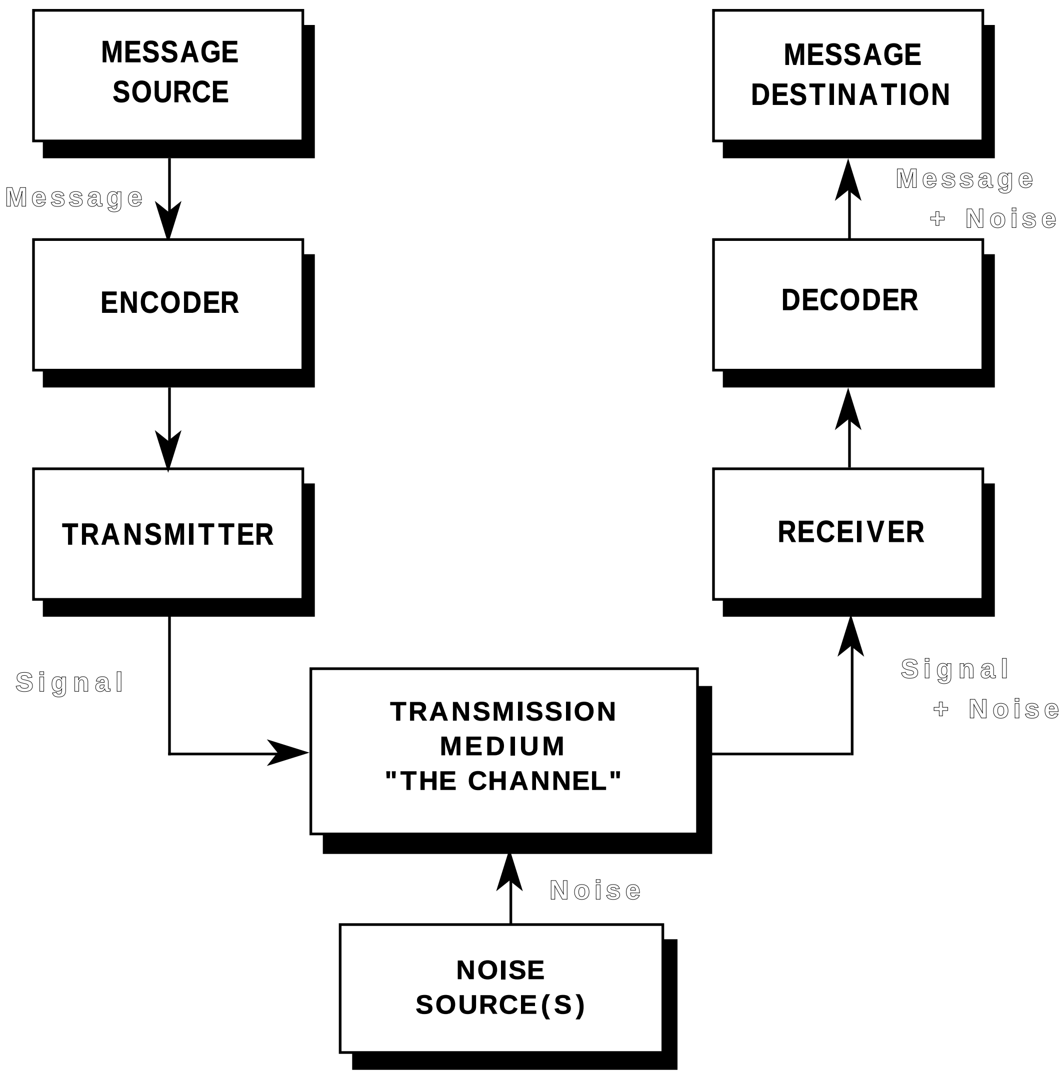
SIGNALS AND CHANNELS

I. GENERAL COMMENTS ON COMMUNICATION PROCESSES:

As a *first cut* in the study of any particular communication process it is useful to examine the diagrammatic representation of that process (a representation which may be attributed to Shannon).



THE *SHANNON* DIAGRAM

By examining the distinct elements of communication processes -- *e.g.*

human thought → speech generation
→ sound propagation → auditory reception → speech perceptio

or

visual scene → video representation - camera → electromagnetic propagation
→ video representation - display → visual perceptio

one may determine how each of these elements limits the overall effectiveness of a given process. In parsing real communication processes, we find each element of this simple diagram may be resolved into a series of subprocesses. For example, in most cases *encoding* involves both source and channel encoding. Here we are particularly interested in the characteristics of ***electromagnetic channels*** and how these characteristics limit our capacity to communicate.

II. ELECTROMAGNETIC CHANNELS -- DIMENSIONALITY/DESCRIPTORS:

"WIRE" v. WIRELESS SYSTEMS

Land telegraphy	Wireless telegraphy
Conventional telephone	Cellular telephones
Intercity coaxial links	Satellite links
Conventional LANs	ALOHA
Fiber optics	Semaphores - Optical telegraphy

GUIDED WAVE PROPAGATION v. FREE SPACE PROPAGATION

Hollow conductor waveguides	Radar beams
Radio signals below 3 MHz	Radio signals above 30 MHz
Dielectric waveguides - optical fiber	Ladar

BASE BAND SIGNALING v. MODULATED CARRIER SIGNAL

Conventional telegraphy	Broadcast radio and TV
Conventional telephony	Multiplexed microwave links
Conventional LANs	Frequency multiplexed LANs

POINT-TO-POINT v. SHARED ACCESS

Conventional telephony	Broadcast radio and TV
Satellite links	LANs and WANs
Fiber optic links	Multiplexed optical links

ANALOG v. DIGITAL

COHERENT v. INCOHERENT

III. CHANNEL ENCODING:

A. AMPLITUDE MODULATION (AM):

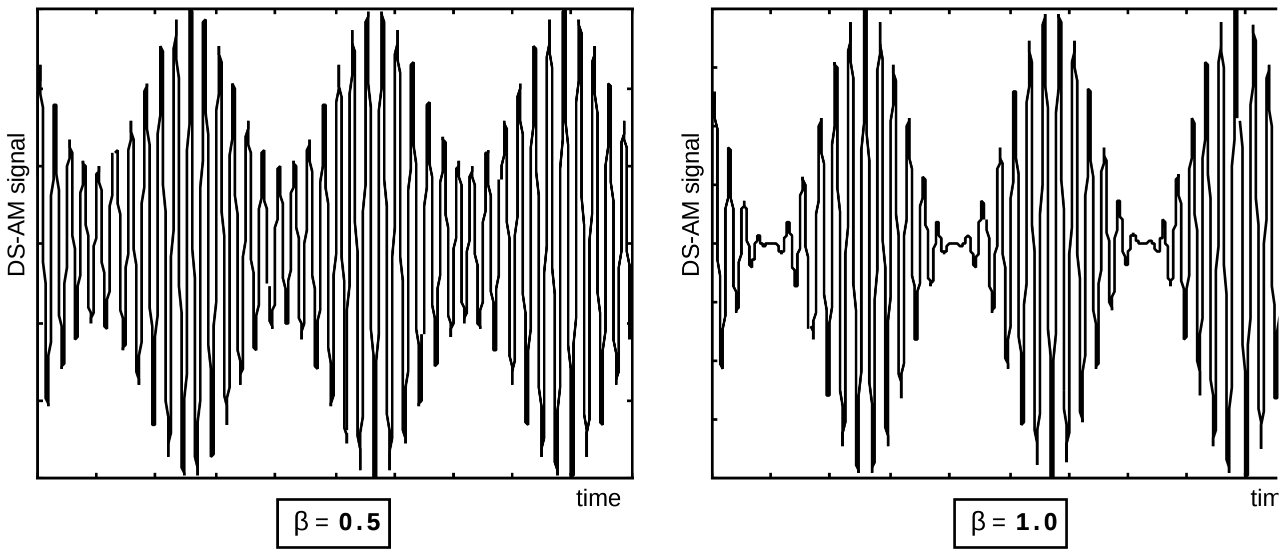
The simplest form of amplitude modulation is called double sideband-amplitude modulation (DS-AM). The DS-AM transmitted signal is of the form

$$\mathrm{v}_s(t) = A_c\left[1 + \mathrm{v}_m(t)\right]\cos(\omega_c t + \varphi_c) \qquad \text{[III-1]}$$

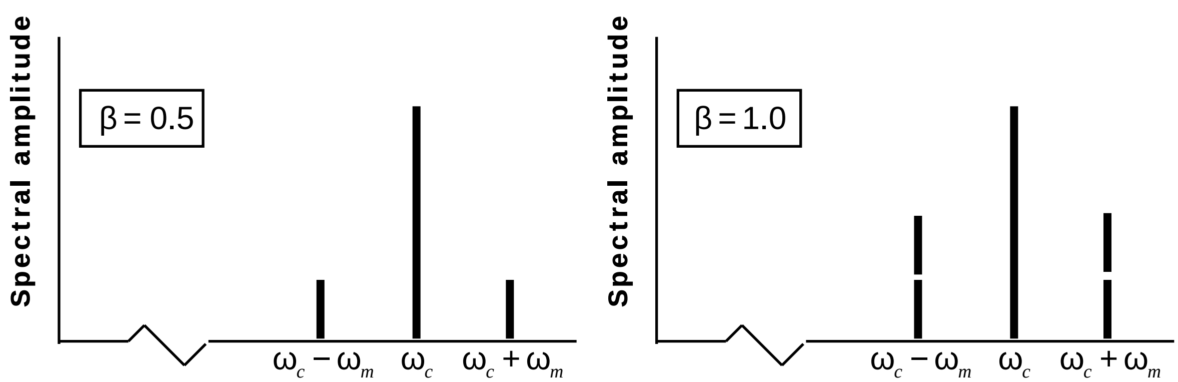
where ω_c is the so called *carrier frequency* and $v_m(t)$ is a temporal representation of the message to be communicated. In particular, for $\mathrm{v}_m(t) = A_m \cos(\omega_m t)$, which may be interpreted as a single Fourier component of the message, the DS-AM signal becomes

$$\begin{aligned}
\mathrm{v}_s(t) &= \cos(\omega_c t)\left[A_c + A_m \cos(\omega_m t)\right] = A_c \cos(\omega_c t)\left[1 + \beta \cos(\omega_m t)\right] \\
&= A_c \left\{\cos(\omega_c t) + \frac{\beta}{2}\cos\left[(\omega_c + \omega_m)t\right] + \frac{\beta}{2}\cos\left[(\omega_c - \omega_m)t\right]\right\} \qquad . \qquad \text{[III -2]} \\
&= \text{"carrier"} + \text{"upper sideband"} + \text{"lower sideband"}
\end{aligned}$$

TEMPORAL REPRESENTATIONS OF DSB-AM SIGNALS



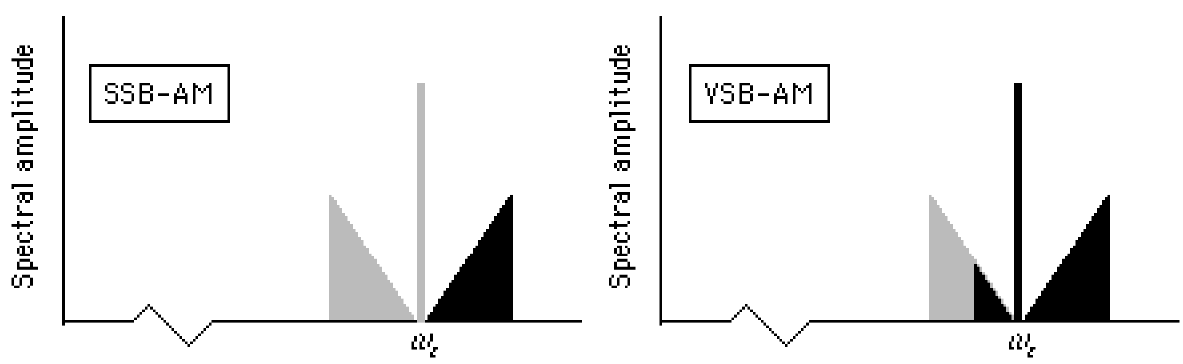
SPECTRAL REPRESENTATIONS OF DSB-AM SIGNALS



Thus the complete spectrum of the of the message is mapped into two, **redundant** sidebands symmetrically disposed with respect to the carrier frequency.

As is demonstrated below, the decoding of DSB-AM signals involves the least complexity, but the redundancy of the information carried by the two sidebands implies an inherent inefficiency in terms of both radiated power and frequency space utilization. In most practical AM systems the carrier and/or some part of one sideband is suppressed. Obviously, the Fourier amplitude at the carrier frequency carries no information so that the double sideband, suppressed carrier, amplitude modulation (DSB-SC-AM) format has enhanced power efficiency. In single sideband amplitude modulation (SSB-AM) format, both the carrier and one complete sideband are suppressed. Thus, SSB-AM is the most efficient in terms of radiated power and frequency space utilization, but involves the greatest complexity in receiver design. A compromise is found in vestigial sideband amplitude modulation (VSB-AM) where a major portion of one sideband is suppressed. VSB-AM is used in NTSC broadcast TV standard where 70% of the lower sideband is suppressed. Schematic spectral representations of typical SSB-AM and VSB-AM signals is sketch below.

SPECTRAL REPRESENTATIONS OF SSB-AM AND VSB SIGNALS



B. ANGLE MODULATION (PM AND FM):

Angle encoded signals have the general form

$$v_s(t) = A_s \cos\theta_s(t) \qquad \text{[III-3]}$$

For **phase modulation** (PM), in general the *instantaneous frequency* is given by

$$\theta_s(t) = \omega_c t + \varphi_c + K_{pm}\, v_m(t). \qquad \text{[III-4]}$$

where K_{pm} is called the *PM deviation constant*. In the particular for sinusoidal modulation -- *i.e.* $v_m(t) = A_m \cos(\omega_m t)$

$$v_s(t) = A_c \cos\left[\omega_c t + \varphi_c + K_{pm} A_m \cos(\omega_m t)\right] = A_c \cos\left[\omega_c t + \varphi_c + \beta_{pm} \cos(\omega_m t)\right] \qquad \text{[III-5]}$$

where β_{pm} is called the *PM modulation index* .

For **frequency modulation** (FM), in general the *instantaneous frequency* is given by

$$\frac{d\theta_s(t)}{dt} = \dot{\theta}_s(t) = \omega_c + K_{fm}\, v_m(t) \qquad \text{[III-6a]}$$

so that
$$\theta_s(t) = \omega_c t + \varphi_c + K_{fm} \int_0^t v_m(t')\, dt' \qquad \text{[III-6b]}$$

where K_{fm} is called the *FM deviation constant*. In particular, for the sinusoidal modulation $v_m(t) = A_m \cos(\omega_m t)$

$$v_s(t) = A_c \cos\left[\omega_c t + \varphi_c + \frac{K_{fm} A_m}{\omega_m} \sin(\omega_m t)\right] = A_c \cos\left[\omega_c t + \varphi_c + \beta_{fm} \sin(\omega_m t)\right] \qquad \text{[III-7]}$$

where β_{fm} is called the *FM modulation index* .[1]

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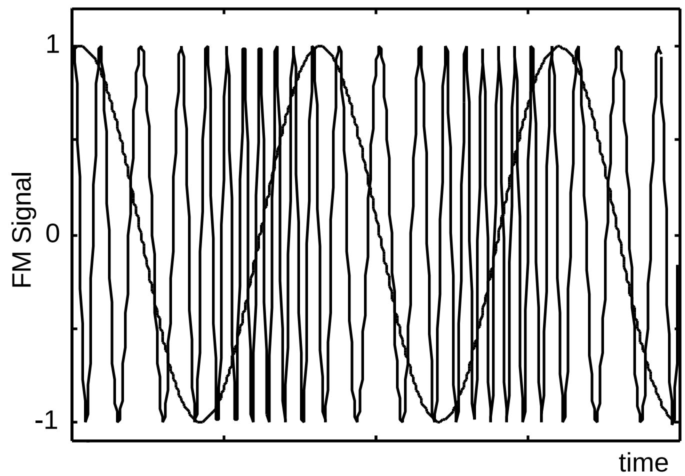
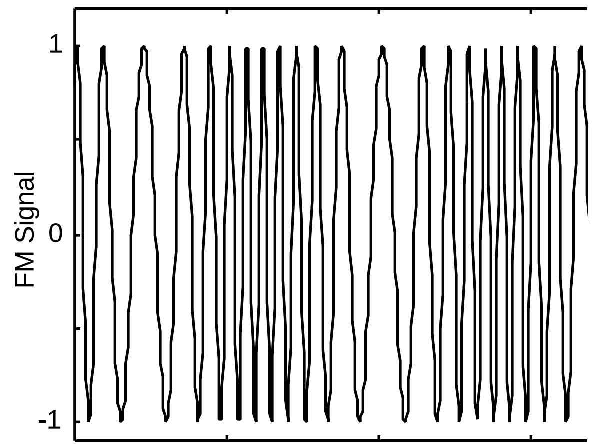
1 In the general case the indices of modulation are given by

TEMPORAL REPRESENTATIONS OF FM SIGNALS



FM Signal with $\beta_{fm} = 5$
(includes an overlay of modulation)



FM Signal with $\beta_{fm} = 5$
(without overlay)

The spectral representation of angular modulation signals is a bit more complicated than the representations of amplitude modulated signals discussed above. Equation [III-7] may be rewritten as

$$v_s(t) = A_c \cos(\omega_c t + \varphi_c) \cos[\beta_{fm} \sin(\omega_m t)] - A_c \sin(\omega_c t + \varphi_c) \sin[\beta_{fm} \sin(\omega_m t)] \ . \quad \text{[III-8]}$$

So that in the limit of small modulation -- *i.e.* for so called *narrowband FM* [2]

$$\beta_{pm} = K_{pm} \max[v_m(t)] \text{ and } \beta_{fm} = \frac{K_{fm} \max[v_m(t)]}{W}$$

where W is the bandwidth of the message signal $v_m(t)$.

2 Going beyond the narrowband limitation, it can be shown that

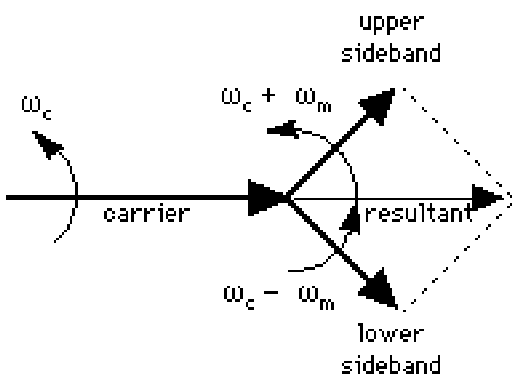
$$\cos[\omega_c t + \beta_{fm} \sin(\omega_m t)] = \sum_{n=-\infty}^{+\infty} J_n(\beta_{fm}) \cos[(\omega_c + n\omega_m) t]$$

$$v_s(t) \approx A_c \cos(\omega_c t + \varphi_c) - A_c\, \beta_{fm} \sin(\omega_c t + \varphi_c) \sin(\omega_m t) \; . \qquad \text{[III-9a]}$$

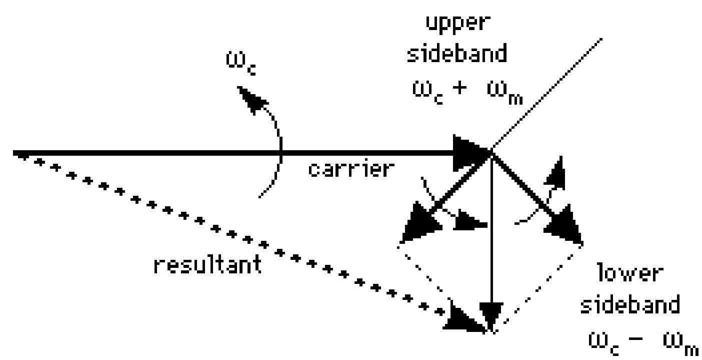
or

$$v_s(t) \approx A_c \left\{ \cos(\omega_c t + \varphi_c) + \frac{\beta_{fm}}{2} \cos[(\omega_c + \omega_m) t + \varphi_c] - \frac{\beta_{fm}}{2} \cos[(\omega_c - \omega_m) t + \varphi_c] \right\} . \qquad \text{[III-9b]}$$

PHASOR REPRESENTATIONS OF AM AND FM SIGNALS



A DSB-AM Signal



A narrowband FM Signal

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where the $J_n(\beta_{fm})$ is the nth order Bessel function of the first kind. Thus, $J_n(\beta_{fm})$ is the value of the relative amplitude of the nth sideband. For example,

$$\left[J_0(1) = 0.765, J_{\pm1}(1) = \pm 0.440, J_{\pm2}(1) = 0.115, J_{\pm3}(1) = \pm 0.020, J_{\pm4}(1) = \pm 0.002, \ldots \right]$$

In general, the effective bandwidth of angle modulated signals -- *i.e.* the frequency range which contains 98% of the signal power -- is given by the so called Carson rule -- *i.e.*

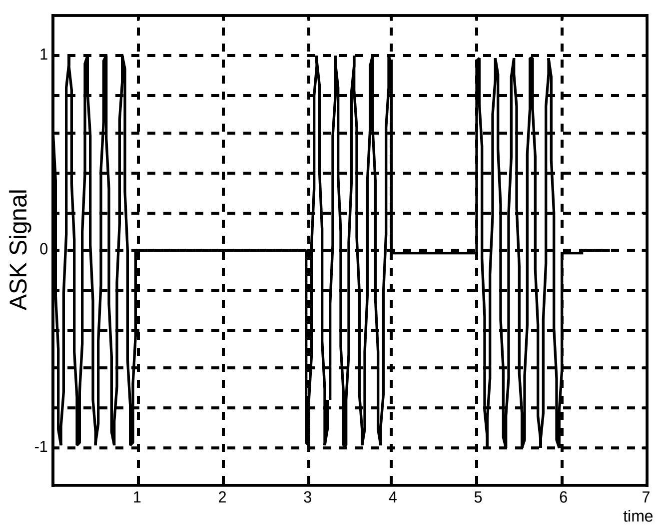
$$\text{effective bandwidth} = 2\, f_m\, (1 + \beta) \, .$$

C. PULSE MODULATIONS (PAM, PPM, AND PFM):

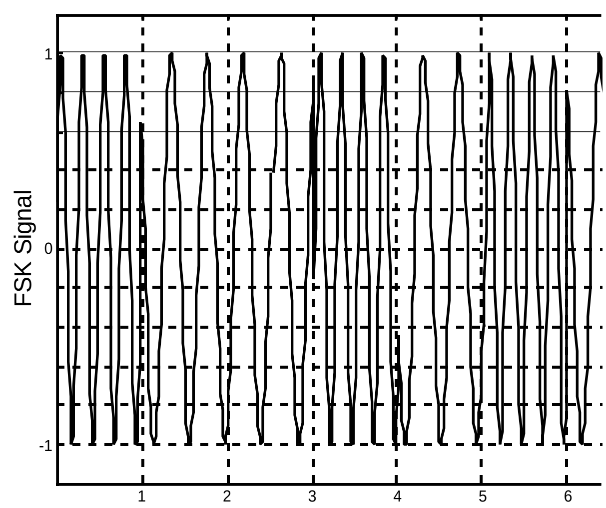
In digital communication the bit steam associated with the data stream derived from the message source may also be encoded in modulated carrier formats -- *viz*.

ANALOG ENCODING		DIGITAL ENCODING
Amplitude modulation (AM)	⇔	Pulse amplitude modulation (PAM) or Amplitude shift keying (ASK),
Phase modulation (PM)	⇔	Pulse phase modulation (PPM) or Phase shift keying (PSK),
Frequency modulation (FM)	⇔	Pulse frequency modulation (PFM) or Frequency shift keying (FSK),

TEMPORAL REPRESENTATIONS OF ASK AND FSK SIGNALS



ASK encoding of a given bit stream



FSK encoding of the same bit stream

IV. MODULATORS AND DEMODULATORS

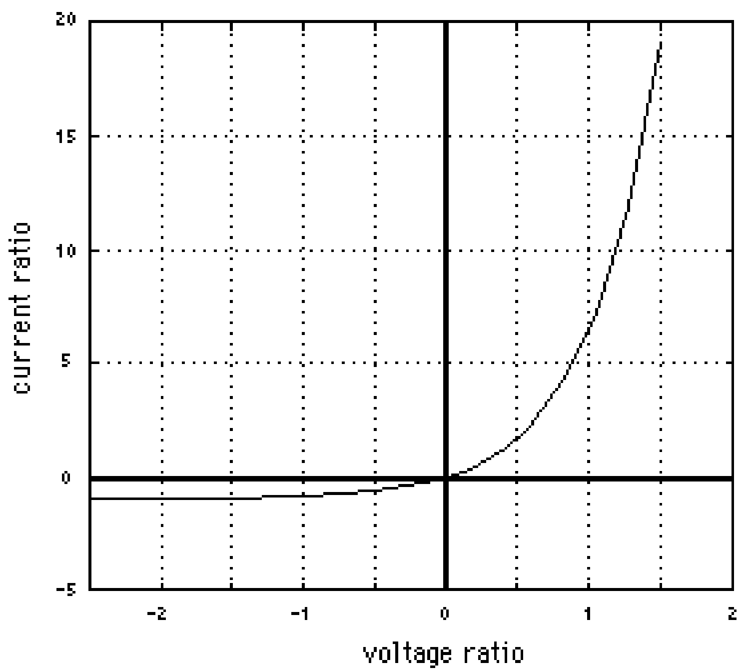
The dictum: *All modulators and demodulator make use of some sort of nonlinear operation!*

Let us explore some particular examples to demonstrate this fundamental truth.

A. DSB-AM MODULATORS:

Perhaps the simplest class of amplitude modulators (and demodulators as well) utilize, in some way, the nonlinear characteristics of diodes. As you know, the iv-characteristic of solid state diodes is reasonably well represented by the expression

$$\mathrm{i}(\mathrm{v}) = \mathrm{i}_0\left[\exp(\mathrm{v}/\mathrm{v}_0) - 1\right] \qquad \text{[IV-1]}$$



where i_0 and v_0 are constants. To see how modulation is achieved, it is useful to expand this iv-characteristic in terms of an infinite series -- *viz*.

$$i(v) = i_0\left[\sum_{n=0}^{\infty}\frac{(v/v_0)^n}{n!} - 1\right] = i_0 \sum_{n=1}^{\infty}\frac{(v/v_0)^n}{n!} = i_0\left[(v/v_0) + \frac{1}{2}(v/v_0)^2 + \frac{1}{6}(v/v_0)^3 + \cdots\right]. \quad \text{[IV-2}$$

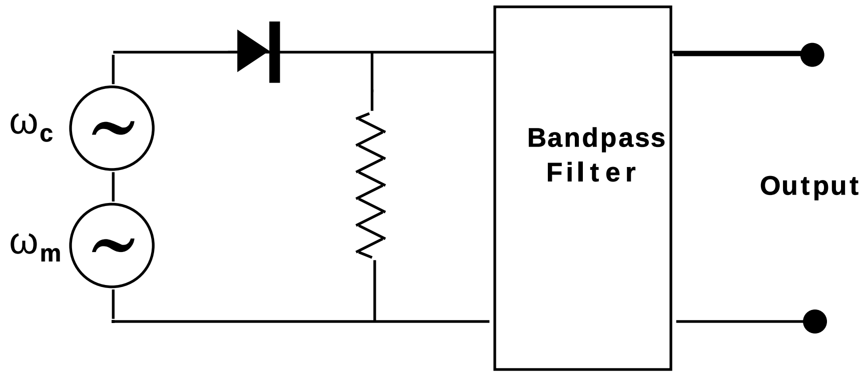
Thus, if the voltage $v = v_c + v_m$ is impressed across the diode, the current through the diode is given by

$$i(v) = i_0\left[\left(\frac{v_c + v_m}{v_0}\right) + \frac{1}{2}\left(\frac{v_c + v_m}{v_0}\right)^2 + \frac{1}{6}\left(\frac{v_c + v_m}{v_0}\right)^3 + \cdots\right]. \quad \text{[IV-3}$$

The second term is includes the crucial product term -- *i.e.* $\left(i_0/v_0^{\,2}\right)\left(v_c\ \ v_m\right)$ -- which generates an AM component in the current through the diode. But how do we get rid of the unwanted the consequence of all the other terms in the expansion? The answer to this critical question lies in the great art of filter synthesis. Examination of Equation [IV-3] shows that when $v_c(t) = A_c\cos(\omega_c t)$ and $v_m(t) = A_m\cos(\omega_m t)$

the term…	**generates**	**frequency components at…**
$\left(\frac{v_c + v_m}{v_0}\right)$		ω_c and ω_m,
$\left(\frac{v_c + v_m}{v_0}\right)^2$		0, $2\omega_c$, $2\omega_m$ and $\omega_c \pm \omega_m$,
$\left(\frac{v_c + v_m}{v_0}\right)^3$		ω_c, ω_m, $3\omega_c$, $3\omega_m$, $2\omega_c \pm \omega_m$ and $\omega_c \pm 2\omega_m$,
$\left(\frac{v_c + v_m}{v_0}\right)^4$		0, $2\omega_c$, $2\omega_m$, $4\omega_c$, $4\omega_m$, , $\omega_c \pm 3\omega_m$, $3\omega_c \pm \omega_m$ and $\omega_c \pm \omega_m$
………		………

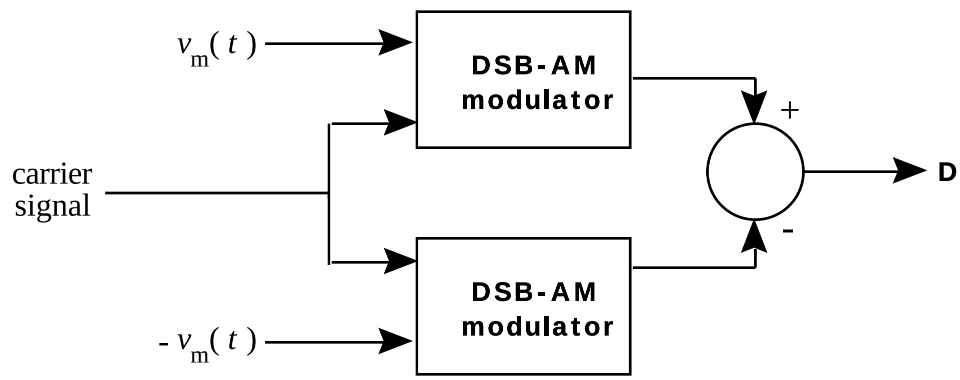
Simple Power Law Modulator:



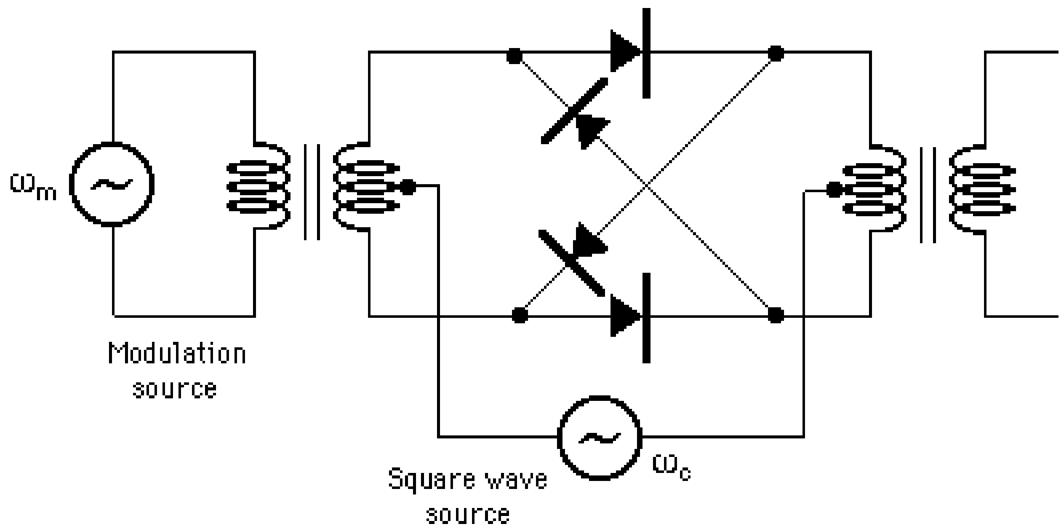
B. BALANCED MODULATORS - *i.e.* DSB-SC-AM ENCODERS:

Power law modulation will always produce a DSM-AM signal -- *i.e.* a carrier signal plus sidebands. To suppress the carrier one need a "pure" device which is called a balanced modulator.

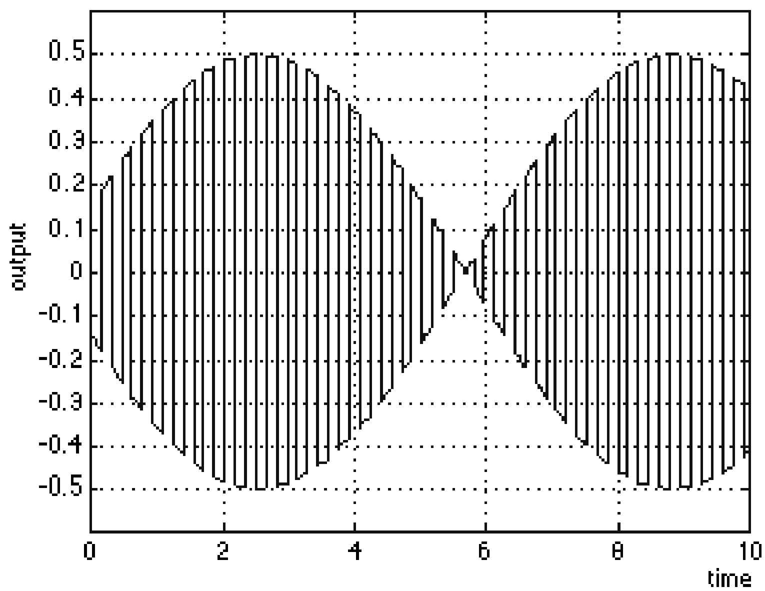
Simple Balanced Modulator:



Switching or Ring Modulator:



Notice that when the square wave is positive the top and bottom diodes conduct and the modulating current flows *ccw* through the output transformer: when it is positive the diodes in the crossarms conduct and the modulating current flows *cw*. Thus, the output signal is the product of the square wave times the modulating signal. Obviously the high frequency components associated with the square wave have to be filtered out to obtain DSB-SC-AM.



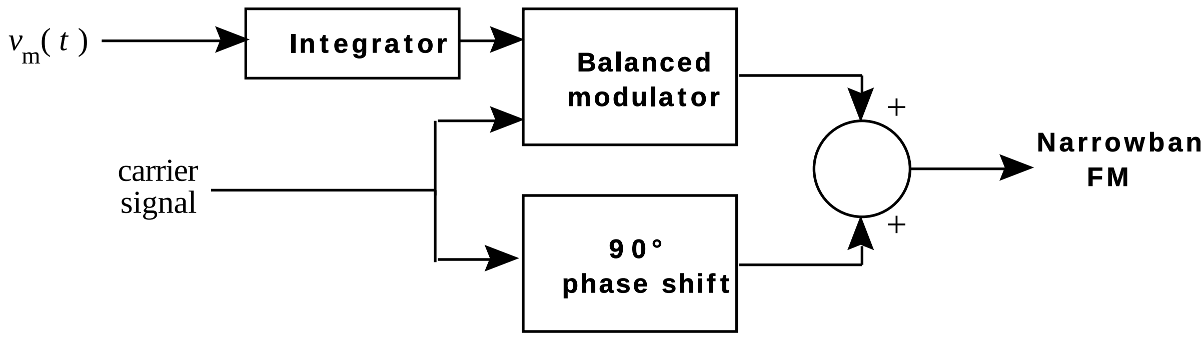
Since it difficult to build sharp filters at high frequencies, SSB-AM is commonly produced by a two step process. In the first step, the information signal is mixed with a relatively low carrier frequency where sharply tuned filters are feasible. At this low frequency, one of the sidebands is stripped off and mixed with a high frequency carrier. The resulting sidebands are separated by the initial low carrier frequency and filtering is easily accomplished.

C. DSB-AM DEMODULATORS:

Again the simplest class of amplitude demodulators utilize, in some way, the nonlinear characteristics of diodes. When, for example, a DSB-AM signal -- *i.e.* Equation [III -2] -- is applied across a diode, the $\left[\left(v_c + v_m\right)/v_0\right]^2$ term in Equation [IV-3] generates, among other things, diode current components at ω_m.

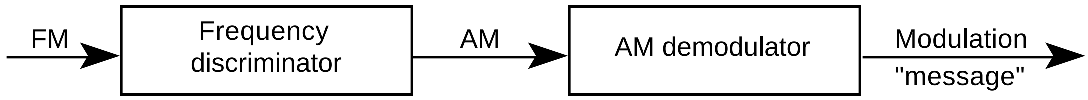
D. FM ENCODERS:

Earlier, we showed that "narrowband FM" -- *i.e.* small β FM -- is equivalent to DSB-SC-AM in quadrature with the carrier signal. Thus, Armstrong's FM encoder:

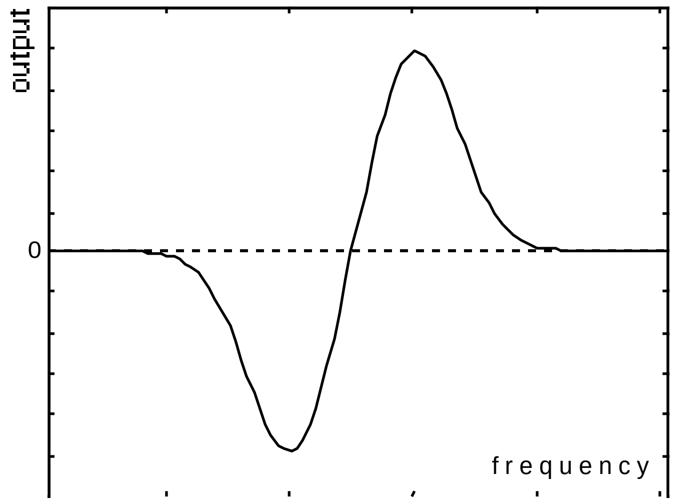


E. FM DECODERS:

Tne following block diagram illustrates the functional operation of one important class of FM decoders:



Tne key element in this class of demodulators is a so called ***frequency discriminator*** -- *i.e.* a device which converts frequency fluctuations into amplitude fluctuations. The characteristics of such a device is depicted below.



V. MEASURING INFORMATION

Consider an information source with ordered outputs:

$$a_1, a_2, a_3, a_4, \ldots a_i, \ldots a_N$$

where a_1 is most-likely and a_N is least-likely -- *e.g.* a_i is the weather condition and air pollution level in a given city and a certain day. The "measure of information" must satisfy the following conditions:

- The information content of output a_i depends only on the probability of a_i -- *i.e.* p_i -- and not on the value of a_i. We denote this function by $I(p_i)$ and call it *self-information.*
- Self-information is a continuous function of p_i.
- Self-information is a decreasing function of p_i.

- If $p_i = p_{(i\,1)}\, p_{(i\,2)}$, then $I(p_i) = I\left(p_{(i\,1)}\right) + I\left(p_{(i\,2)}\right)$.

Only function that satisfies these properties is $$I(p_i) = -\log(p_i)$$

Therefore the information revealed by a particular source output is the "weighted" average of the self-information of the various outputs --

$$H(X) = -\sum_{i=1}^{N} p_i \log(p_i) = \sum_{i=1}^{N} \log\left(\frac{1}{p_i}\right)^{p_i} \qquad \text{[V-1]}$$

which is usually called the *entropy* of the source.

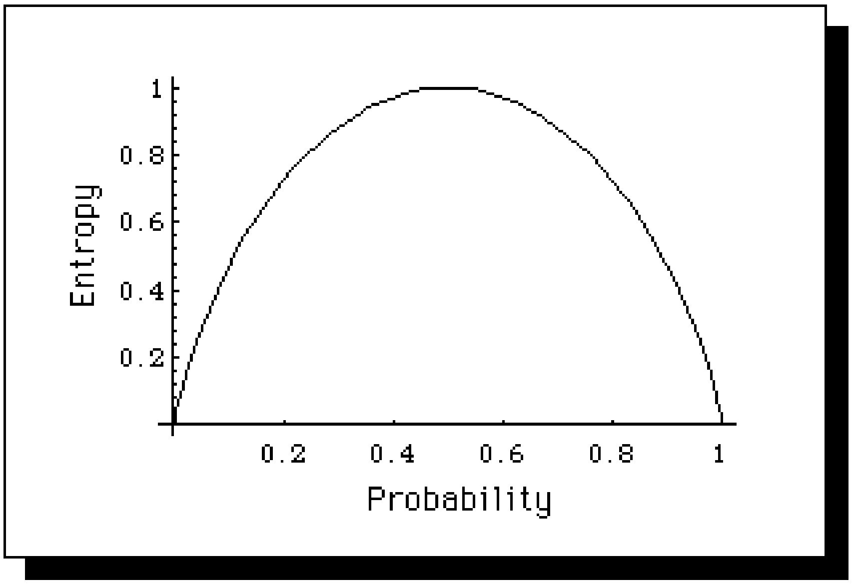
EXAMPLES OF ENTROPY CALCULATIONS

- Consider a random variable uniformly distributed over 32 outcomes:

$$H(X) = -\sum_{i=1}^{32} p(i) \log_2 p(i) = -\sum_{i=1}^{32} \frac{1}{32} \log_2 \frac{1}{32} = 5\text{bits}$$

- Consider a discrete memoryless information source with binary output alphabet with respective probabilities p and (1 - p):

$$H = -p \log_2(p) - (1-p) \log_2(1-p)$$



That is, equal probabilities implies maximum entropy -- A general truth!

- Consider a source with a bandwidth of 4 kHz which is sampled at the Nyquist rate (*i.e.* 8,000 samples per second). The resulting sample sequence can be approximated as a discrete memoryless information source with an output alphabet $\{-2, -1, 0, +1, +2\}$ and with has corresponding probabilities $\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}\right\}$.

$$H = -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{8}\log_2\left(\frac{1}{8}\right) - \frac{1}{16}\log_2\left(\frac{1}{16}\right) - \frac{1}{16}\log_2\left(\frac{1}{16}\right)$$
$$= (15/8) \text{ bits per sample}$$

Therefore the source produces information at a rate of 15 kilobits per second.

- Consider an source which generates five letters **{A, B, C, D, E}** with equal probabilities.

  Typical message:

BDCBCECCCADCBDDAAECEEAABBDAEECACEEBAEECBCEAD

$$H = 5\left[-\frac{1}{5}\log_2\left(\frac{1}{5}\right)\right] = \frac{\log_{10}(5)}{\log_{10}(2)} = 2.32 \text{ bits per sample}$$

- Consider an source which generates five letters **{A, B, C, D, E}** with probabilities **{0.4, 0.1, 0.2, 0.2, 0.1}.**
  Typical message:

  AAACDCBDCEAADADACEDAEADCABEDADDCECAAAAAD

$$H = \left[\log_{10}(2)\right]^{-1}\left[-0.4\log_{10}(0.4) - 0.1\log_{10}(0.1) - 0.2\log_{10}(0.2) - 0.2\log_{10}(0.2) - 0.1\log_{10}(0.1)\right]$$
$$= 2.12 \text{ bits per sample}$$

- Consider a horse race with eight horses taking part. Suppose that the probabilities of each horse winning are given by [3]

$$p(1) = \frac{1}{2},\; p(2) = \frac{1}{4},\; p(3) = \frac{1}{8},\; p(4) = \frac{1}{16}, \text{ and } p(5) = p(6) = p(7) = p(8) = \frac{1}{64}.$$

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[3] **ALTERNATE CODING SCHEMES FOR HORSE RACE RESULTS**

1	000	0
2	001	10
3	010	110
4	011	1110
5	100	111100
6	101	111101
7	110	111110
8	111	111111
Avererage number of bits used	3 bits	2 bits

In general, the entropy of a random variable is a lower bound on the number of bits required to represent the random variable and on the average number of questions needed to identify the variable in a game of "twenty questions."

$$H(X) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{8}\log_2\frac{1}{8} - \frac{1}{16}\log_2\frac{1}{16} - 4\,\frac{1}{64}\log_2\frac{1}{64} = 2 \text{ bits}$$

- Consider an source which generates three letters **{A, B, C}** of an artificial language specified by the probability $p_i(j)$ that the letter *i* is followed by the letter *j* -- *viz*.

$p_i(j)$	**A**	**B**	**C**
A	0	4/5	1/5
B	1/2	1/2	0
C	1/2	2/5	1/10

or the probability $p(i,j)$ of the **digram** "*ij*" -- *viz*.

$p(i,j)$	**A**	**B**	**C**
A	0	4/15	1/15
B	8/27	8/27	0
C	1/27	4/135	1/135

i	$p(i)$
A	9/27
B	16/27
C	2/17

These various probabilities are related by the following expression:

$$p(i) = \sum_j p(i,j) = \sum_j p(j,i) = \sum_j p(j)\,p_j(i) \qquad \text{[V-2]}$$

Typical message:

ABBABABABABABABBBABBBBBBABABAB
ABABBBACACABBABBBBABBABACBBBABA