I. GENERAL COMMENTS ON COMMUNICATION PROCESSES:
As a first cut in the study of any particular communication process it is useful to examine the diagrammatic representation of that process (a representation which may be attributed to Shannon).

THE SHANNON DIAGRAM
By examining the distinct elements of communication processes -- e.g. 

human thought $\rightarrow$ speech generation $\rightarrow$ sound propagation $\rightarrow$ auditory reception $\rightarrow$ speech perception

or

visual scene $\rightarrow$ video representation $\rightarrow$ camera $\rightarrow$ electromagnetic propagation $\rightarrow$ video representation $\rightarrow$ visual perception

one may determine how each of these elements limits the overall effectiveness of a given process. In parsing real communication processes, we find each element of this simple diagram may be resolved into a series of subprocesses. For example, in most cases encoding involves both source and channel encoding. Here we are particularly interested in the characteristics of electromagnetic channels and how these characteristics limit our capacity to communicate.

II. ELECTROMAGNETIC CHANNELS -- DIMENSIONALITY/DESCRIPTEORS:

“WIRE” v. WIRELESS SYSTEMS

<table>
<thead>
<tr>
<th>Guided Wave Propagation v. Free Space Propagation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land telegraphy</td>
</tr>
<tr>
<td>Conventional telephone</td>
</tr>
<tr>
<td>Intercity coaxial links</td>
</tr>
<tr>
<td>Conventional LANs</td>
</tr>
<tr>
<td>Fiber optics</td>
</tr>
<tr>
<td>Guided Wave Propagation</td>
</tr>
<tr>
<td>Hollow conductor waveguides</td>
</tr>
<tr>
<td>Radio signals below 3 MHz</td>
</tr>
<tr>
<td>Dielectric waveguides - optical fiber</td>
</tr>
</tbody>
</table>


**Base Band Signaling v. Modulated Carrier Signal**

- Conventional telegraphy: Broadcast radio and TV
- Conventional telephony: Multiplexed microwave links
- Conventional LANs: Frequency multiplexed LANs

**Point-to-Point v. Shared Access**

- Conventional telephony: Broadcast radio and TV
- Satellite links: LANs and WANs
- Fiber optic links: Multiplexed optical links

**Analog v. Digital**

**Coherent v. Incoherent**

### III. Channel Encoding:

**A. Amplitude Modulation (AM):**

The simplest form of amplitude modulation is called double sideband-amplitude modulation (DS-AM). The DS-AM transmitted signal is of the form

\[ v_s(t) = A_c [1 + v_m(t)] \cos(\omega_c t + \phi_c) \]  \[ \text{[III-1]} \]

where \( \omega_c \) is the so called *carrier frequency* and \( v_m(t) \) is a temporal representation of the message to be communicated. In particular, for \( v_m(t) = A_m \cos(\omega_m t) \), which may be interpreted as a single Fourier component of the message, the DS-AM signal becomes

\[ v_s(t) = \cos(\omega_c t) \left[ A_c + A_m \cos(\omega_m t) \right] = A_c \cos(\omega_c t) \left[ 1 + \beta \cos(\omega_m t) \right] \]

\[ = A_c \left\{ \cos(\omega_c t) + \frac{\beta}{2} \left[ \cos(\omega_c + \omega_m) t + \cos(\omega_c - \omega_m) t \right] \right\} \]  \[ \text{[III -2]} \]

\[ = \text{"carrier" + "upper sideband" + "lower sideband"} \]
Thus the complete spectrum of the message is mapped into two, **redundant** sidebands symmetrically disposed with respect to the carrier frequency.
As is demonstrated below, the decoding of DSB-AM signals involves the least complexity, but the redundancy of the information carried by the two sidebands implies an inherent inefficiency in terms of both radiated power and frequency space utilization. In most practical AM systems the carrier and/or some part of one sideband is suppressed. Obviously, the Fourier amplitude at the carrier frequency carries no information so that the double sideband, suppressed carrier, amplitude modulation (DSB-SC-AM) format has enhanced power efficiency. In single sideband amplitude modulation (SSB-AM) format, both the carrier and one complete sideband are suppressed. Thus, SSB-AM is the most efficient in terms of radiated power and frequency space utilization, but involves the greatest complexity in receiver design. A compromise is found in vestigial sideband amplitude modulation (VSB-AM) where a major portion of one sideband is suppressed. VSB-AM is used in NTSC broadcast TV standard where 70% of the lower sideband is suppressed. Schematic spectral representations of typical SSB-AM and VSB-AM signals is sketch below.

**Spectral Representations of SSB-AM and VSB Signals**

![Spectral Amplitude Diagrams](image)

**B. Angle Modulation (PM and FM):**

Angle encoded signals have the general form
For phase modulation (PM), in general the instantaneous frequency is given by

\[ \theta_s(t) = \omega_c t + \phi_c + K_{pm} v_m(t). \]  

[III-4]

where \( K_{pm} \) is called the PM deviation constant. In the particular for sinusoidal modulation -- i.e. \( v_m(t) = A_m \cos(\omega_m t) \)

\[ v_s(t) = A_c \cos(\omega_c t + \phi_c + K_{pm} A_m \cos(\omega_m t)) = A_c \cos(\omega_c t + \phi_c + \beta_{pm} \cos(\omega_m t)) \]  

[III-5]

where \( \beta_{pm} \) is called the PM modulation index.

For frequency modulation (FM), in general the instantaneous frequency is given by

\[ \frac{d\theta_s(t)}{dt} = \dot{\theta}_s(t) = \omega_c + K_{fm} v_m(t) \]  

[III-6a]

so that

\[ \theta_s(t) = \omega_c t + \phi_c + K_{fm} \int_0^t v_m(t') dt' \]  

[III-6b]

where \( K_{fm} \) is called the FM deviation constant. In particular, for the sinusoidal modulation \( v_m(t) = A_m \cos(\omega_m t) \)

\[ v_s(t) = A_c \cos\left[ \omega_c t + \phi_c + \frac{K_{fm} A_m}{\omega_m} \sin(\omega_m t) \right] = A_c \cos\left[ \omega_c t + \phi_c + \beta_{fm} \sin(\omega_m t) \right] \]  

[III-7]

where \( \beta_{fm} \) is called the FM modulation index.  

1 In the general case the indices of modulation are given by
The spectral representation of angular modulation signals is a bit more complicated than the representations of amplitude modulated signals discussed above. Equation [III-7] may be rewritten as

\[ v_s(t) = A_c \cos(\omega_c t + \phi_c) \cos[\beta_{fm} \sin(\omega_m t)] - A_c \sin(\omega_c t + \phi_c) \sin[\beta_{fm} \sin(\omega_m t)]. \]  \[ \text{[III-8]} \]

So that in the limit of small modulation -- i.e. for so called narrowband FM ²

\[ \beta_{pm} = K_{pm} \max[v_m(t)] \quad \text{and} \quad \beta_{fm} = \frac{K_{fm}}{W} \max[v_m(t)] \]

where \( W \) is the bandwidth of the message signal \( v_m(t) \).

² Going beyond the narrowband limitation, it can be shown that

\[ \cos[\omega_c t + \beta_{fm} \sin(\omega_m t)] = \sum_{n=-\infty}^{\infty} J_n(\beta_{fm}) \cos[(\omega_c + n\omega_m) t] \]

\[ \text{where} \quad J_n(\beta_{fm}) \text{is the Bessel function of the first kind.} \]
**Signals and Channels**

\[ v_s(t) = A_c \cos(\omega_c t + \phi_c) - A_c \beta_{fm} \sin(\omega_c t + \phi_c) \sin(\omega_m t) . \] \[ \text{[III-9a]} \]

or

\[ v_s(t) = A_c \left\{ \cos(\omega_c t + \phi_c) \right. \\
+ \left. \frac{\beta_{fm}}{2} \cos[(\omega_c + \omega_m) t + \phi_c] - \frac{\beta_{fm}}{2} \cos[(\omega_c - \omega_m) t + \phi_c] \right\} . \] \[ \text{[III-9b]} \]

**Phasor Representations of AM and FM Signals**

- A DSB-AM Signal
- A narrowband FM Signal

where the \( J_n(\beta_{fm}) \) is the \( n \)th order Bessel function of the first kind. Thus, \( J_n(\beta_{fm}) \) is the value of the relative amplitude of the \( n \)th sideband. For example,

\[
\begin{align*}
J_0(1) &= 0.765, \\
J_{\pm1}(1) &= \pm0.440, \\
J_{\pm2}(1) &= 0.115, \\
J_{\pm3}(1) &= \pm0.020, \\
J_{\pm4}(1) &= \pm0.002,
\end{align*}
\]

In general, the effective bandwidth of angle modulated signals -- *i.e.* the frequency range which contains 98% of the signal power -- is given by the so called Carson rule -- *i.e.*

\[	ext{effective bandwidth} = 2 f_m \left(1 + \beta\right).
\]
C. **Pulse Modulations (PAM, PPM, and PFM):**

In digital communication the bit stream associated with the data stream derived from the message source may also be encoded in modulated carrier formats -- *viz.*

<table>
<thead>
<tr>
<th><strong>ANALOG ENCODING</strong></th>
<th><strong>DIGITAL ENCODING</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude modulation (AM)</td>
<td>Pulse amplitude modulation (PAM) or Amplitude shift keying (ASK),</td>
</tr>
<tr>
<td>Phase modulation (PM)</td>
<td>Pulse phase modulation (PPM) or Phase shift keying (PSK),</td>
</tr>
<tr>
<td>Frequency modulation (FM)</td>
<td>Pulse frequency modulation (PFM) or Frequency shift keying (FSK),</td>
</tr>
</tbody>
</table>

**TEMPORAL REPRESENTATIONS OF ASK AND FSK SIGNALS**

![ASK and FSK Signals](image)

ASK encoding of a given bit stream

FSK encoding of the same bit stream
IV. MODULATORS AND DEMODULATORS

The dictum: *All modulators and demodulator make use of some sort of nonlinear operation!*

Let us explore some particular examples to demonstrate this fundamental truth.

A. DSB-AM MODULATORS:

Perhaps the simplest class of amplitude modulators (and demodulators as well) utilize, in some way, the nonlinear characteristics of diodes. As you know, the $iv$-characteristic of solid state diodes is reasonably well represented by the expression

$$i(v) = i_0 \left[ \exp(v/v_0) - 1 \right]$$  \hspace{1cm} [IV-1]
where \( i_0 \) and \( v_0 \) are constants. To see how modulation is achieved, it is useful to expand this \( iv \) characteristic in terms of an infinite series -- viz.

\[
i(v) = i_0 \left[ \sum_{n=0}^{\infty} \frac{(v/v_0)^n}{n!} - 1 \right] = i_0 \sum_{n=1}^{\infty} \frac{(v/v_0)^n}{n!} = i_0 \left[ (v/v_0) + \frac{1}{2} (v/v_0)^2 + \frac{1}{6} (v/v_0)^3 + \cdots \right]. \tag{IV-2}
\]

Thus, if the voltage \( v = v_c + v_m \) is impressed across the diode, the current through the diode is given by

\[
i(v) = i_0 \left[ \left( \frac{v_c + v_m}{v_0} \right) + \frac{1}{2} \left( \frac{v_c + v_m}{v_0} \right)^2 + \frac{1}{6} \left( \frac{v_c + v_m}{v_0} \right)^3 + \cdots \right]. \tag{IV-3}
\]

The second term is includes the crucial product term -- \( i_0/v_0^2 (v_c \cdot v_m) \) -- which generates an AM component in the current through the diode. But how do we get rid of the unwanted consequence of all the other terms in the expansion? The answer to this critical question lies in the great art of filter synthesis. Examination of Equation [IV-3] shows that when \( v_c(t) = A_c \cos(\omega_c t) \) and \( v_m(t) = A_m \cos(\omega_m t) \)

<table>
<thead>
<tr>
<th>the term…</th>
<th>generates frequency components at…</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{v_c + v_m}{v_0} )</td>
<td>( \omega_c ) and ( \omega_m ),</td>
</tr>
<tr>
<td>( \frac{(v_c + v_m)^2}{v_0} )</td>
<td>( 0, 2\omega_c, 2\omega_m ), and ( \omega_c \pm \omega_m ),</td>
</tr>
<tr>
<td>( \frac{(v_c + v_m)^3}{v_0} )</td>
<td>( \omega_c, \omega_m, 3\omega_c, 3\omega_m ),</td>
</tr>
<tr>
<td>( \frac{(v_c + v_m)^4}{v_0} )</td>
<td>( 2\omega_c \pm \omega_m ) and ( \omega_c \pm 2\omega_m ),</td>
</tr>
<tr>
<td>…………</td>
<td>………,…</td>
</tr>
</tbody>
</table>
Simple Power Law Modulator:

\[ \omega_c \]

\[ \omega_m \]

Bandpass Filter

Output

B. BALANCED MODULATORS - *i.e.* DSB-SC-AM ENCODERS:

Power law modulation will always produce a DSM-AM signal -- *i.e.* a carrier signal plus sidebands. To suppress the carrier one need a “pure” device which is called a balanced modulator.

Simple Balanced Modulator:
Switching or Ring Modulator:

Notice that when the square wave is positive the top and bottom diodes conduct and the modulating current flows \( ccw \) through the output transformer: when it is positive the diodes in the crossarms conduct and the modulating current flows \( cw \). Thus, the output signal is the product of the square wave times the modulating signal. Obviously the high frequency components associated with the square wave have to be filtered out to obtain DSB-SC-AM.
Since it difficult to build sharp filters at high frequencies, SSB-AM is commonly produced by a two step process. In the first step, the information signal is mixed with a relatively low carrier frequency where sharply tuned filters are feasible. At this low frequency, one of the sidebands is stripped off and mixed with a high frequency carrier. The resulting sidebands are separated by the initial low carrier frequency and filtering is easily accomplished.

C. DSB-AM DEMODULATORS:
Again the simplest class of amplitude demodulators utilize, in some way, the nonlinear characteristics of diodes. When, for example, a DSB-AM signal -- i.e. Equation [III-2] -- is applied across a diode, the \( \left[(v_c + v_m)/v_0 \right]^2 \) term in Equation [IV-3] generates, among other things, diode current components at \( \omega_m \).
D. FM ENCODERS:
Earlier, we showed that "narrowband FM" -- i.e. small $\beta$ FM -- is equivalent to DSB-SC-AM in quadrature with the carrier signal. Thus, Armstrong's FM encoder:

\[ v_m(t) \rightarrow \text{Integrator} \rightarrow \text{Balanced modulator} \rightarrow 90^\circ \text{phase shift} \rightarrow \text{Narrowband FM} \]

E. FM DECODERS:
The following block diagram illustrates the functional operation of one important class of FM decoders:

\[ \text{FM} \rightarrow \text{Frequency discriminator} \rightarrow \text{AM} \rightarrow \text{AM demodulator} \rightarrow \text{Modulation "message"} \]

The key element in this class of demodulators is a so called frequency discriminator -- i.e. a device which converts frequency fluctuations into amplitude fluctuations. The characteristics of such a device is depicted below.
V. Measuring Information

Consider an information source with ordered outputs:

\[ a_1, a_2, a_3, a_4, \ldots, a_i, \ldots, a_N \]

where \( a_1 \) is most-likely and \( a_N \) is least-likely -- e.g. \( a_i \) is the weather condition and air pollution level in a given city and a certain day. The "measure of information" must satisfy the following conditions:

- The information content of output \( a_i \) depends only on the probability of \( a_i \) -- i.e. \( p_i \) -- and not on the value of \( a_i \). We denote this function by \( I(p_i) \) and call it self-information.
- Self-information is a continuous function of \( p_i \).
- Self-information is a decreasing function of \( p_i \).
• If \( p_i = p_{(i,1)} p_{(i,2)} \), then \( I(p_i) = I(p_{(i,1)}) + I(p_{(i,2)}) \).

Only function that satisfies these properties is \( I(p_i) = -\log(p_i) \)

Therefore the information revealed by a particular source output is the "weighted" average of the self-information of the various outputs --

\[
H(X) = -\sum_{i=1}^{N} p_i \log(p_i) = \sum_{i=1}^{N} \left( \frac{1}{p_i} \right)^{p_i}
\]  

which is usually called the *entropy* of the source.

**Examples of Entropy Calculations**

• Consider a random variable uniformly distributed over 32 outcomes:

\[
H(X) = -\sum_{i=1}^{32} p(i) \log_2 p(i) = -\sum_{i=1}^{32} \frac{1}{32} \log_2 \frac{1}{32} = 5 \text{bits}
\]

• Consider a discrete memoryless information source with binary output alphabet with respective probabilities \( p \) and \( 1 - p \):

\[
H = -p \log_2(p) - (1 - p) \log_2(1 - p)
\]
That is, equal probabilities implies maximum entropy -- A general truth!

- Consider a source with a bandwidth of 4 kHz which is sampled at the Nyquist rate (i.e. 8,000 samples per second). The resulting sample sequence can be approximated as a discrete memoryless information source with an output alphabet \{-2, -1, 0, 1, 2\} and with corresponding probabilities \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{16}\}.

\[
H = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{4} \log_2\left(\frac{1}{4}\right) - \frac{1}{8} \log_2\left(\frac{1}{8}\right) - \frac{1}{16} \log_2\left(\frac{1}{16}\right) - \frac{1}{16} \log_2\left(\frac{1}{16}\right)
\]

= (15/8) bits per sample

Therefore the source produces information at a rate of 15 kilobits per second.

- Consider an source which generates five letters \{A, B, C, D, E\} with equal probabilities.
  Typical message:
Consider an source which generates five letters \{A, B, C, D, E\} with probabilities \{0.4, 0.1, 0.2, 0.2, 0.1\}. Typical message:

\[
\text{AAACDCBCEAADADCEADAEADCEBDEADDCECAAAAAD}
\]

\[
H = \left[ \log_{10}(2) \right]^{-1} \left[ -0.4 \log_{10}(0.4) - 0.1 \log_{10}(0.1) - 0.2 \log_{10}(0.2) - 0.2 \log_{10}(0.2) - 0.1 \log_{10}(0.1) \right]
\]

\[
= 2.12 \text{ bits per sample}
\]

Consider a horse race with eight horses taking part. Suppose that the probabilities of each horse winning are given by \( p(1) = \frac{1}{2}, \ p(2) = \frac{1}{4}, \ p(3) = \frac{1}{8}, \ p(4) = \frac{1}{16}, \ \text{and} \ p(5) = p(6) = p(7) = p(8) = \frac{1}{64} \).

\[
\text{ALTERNATE CODING SCHEMES FOR HORSE RACE RESULTS}
\]

<table>
<thead>
<tr>
<th>( p )</th>
<th>000</th>
<th>001</th>
<th>010</th>
<th>011</th>
<th>100</th>
<th>101</th>
<th>110</th>
<th>111</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
</tbody>
</table>

Average number of bits used

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 bits</td>
<td>2 bits</td>
</tr>
</tbody>
</table>

In general, the entropy of a random variable is a lower bound on the number of bits required to represent the random variable and on the average number of questions needed to identify the variable in a game of "twenty questions."
\[ H(X) = -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{4} \log_2 \frac{1}{4} - \frac{1}{8} \log_2 \frac{1}{8} - \frac{1}{16} \log_2 \frac{1}{16} - \frac{1}{64} \log_2 \frac{1}{64} = 2 \text{ bits} \]

- Consider an source which generates three letters \{A, B, C\} of an artificial language specified by the probability \( p_i(j) \) that the letter \( i \) is followed by the letter \( j \) -- viz.

<table>
<thead>
<tr>
<th>( p_i(j) )</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>4/5</td>
<td>1/5</td>
</tr>
<tr>
<td>B</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1/2</td>
<td>2/5</td>
<td>1/10</td>
</tr>
</tbody>
</table>

or the probability \( p(i, j) \) of the digram "ij" -- viz.

<table>
<thead>
<tr>
<th>( p(i, j) )</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>4/15</td>
<td>1/15</td>
</tr>
<tr>
<td>B</td>
<td>8/27</td>
<td>8/27</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>1/27</td>
<td>4/135</td>
<td>1/135</td>
</tr>
</tbody>
</table>

These various probabilities are related by the following expression:

\[ p(i) = \sum_j p(i, j) = \sum_j p(j, i) = \sum_j p(j)p_i(j) \quad [V-2] \]

Typical message:

ABBABABABABABBBBABBABABABABAB
ABBABABACACABBABBBABBABACBBBABA