ENGINEERING SCIENCES 154

ELECTRONIC DEVICES AND CIRCUITS

SAMPLE FINAL EXAMINATION

FALL TERM 2001-2002

NAME Some Possible Solutions

a. Please answer all of the questions in the spaces provided. If you need additional space, use the backs of the sheets.
b. Partial credit is achievable**, so include all of your calculations and clearly indicate what you are trying to do**.
c. Note that you have **modicum of choice** in the first question.
d. The relative credit assigned to each question is indicated as a ***prudent time allocation.*** That is, there is a possible total credit of **180**.

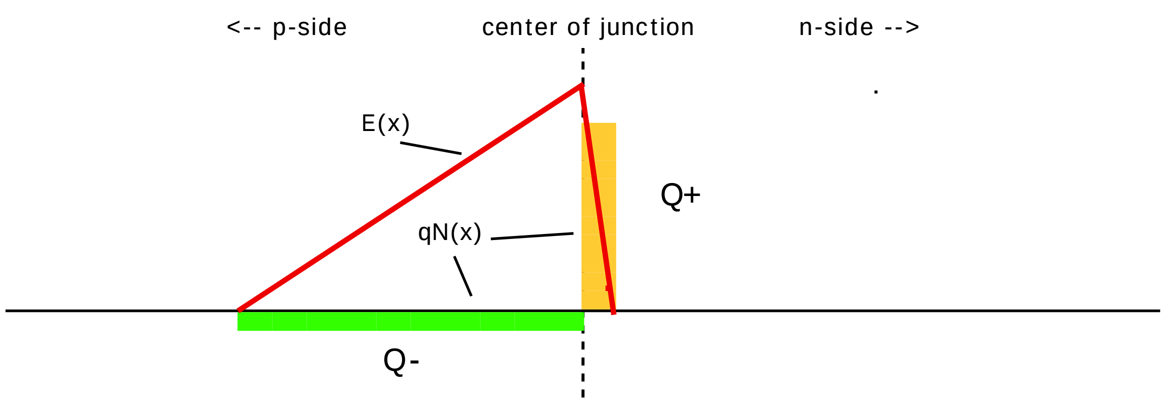
1. (Prudent time allocation = 90 minutes)

   Briefly answer **NINE (9)** of the following **FOURTEEN (14)** questions:

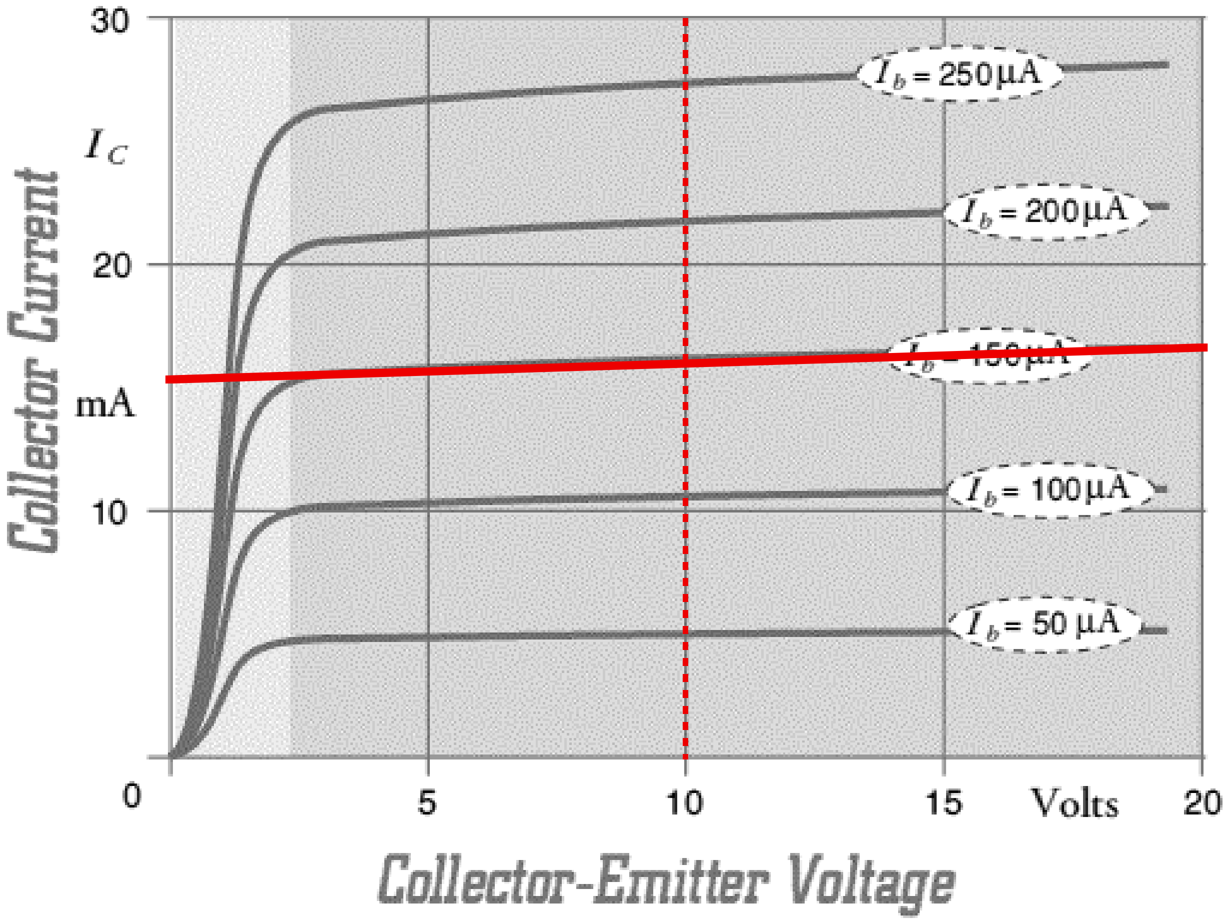
   a. In the space below, plot the **net charge density** (sign and magnitude) **<u>and</u>** the **built-in electric field** as a function of position along a line which intersects a pn homojunction at right angles. The n-side of the junction has a doping level that is 10 times that of the p-side (*i.e.*, $N_D = 10\ N_A$). Label and/or note important features.

See discussion at:

www.deas.harvard.edu/courses/es154/lectures/lecture_2/pn_junction/pn_junction.html#space_charge



b. Suppose that a particular BJT has the following collector current characteristic curve:



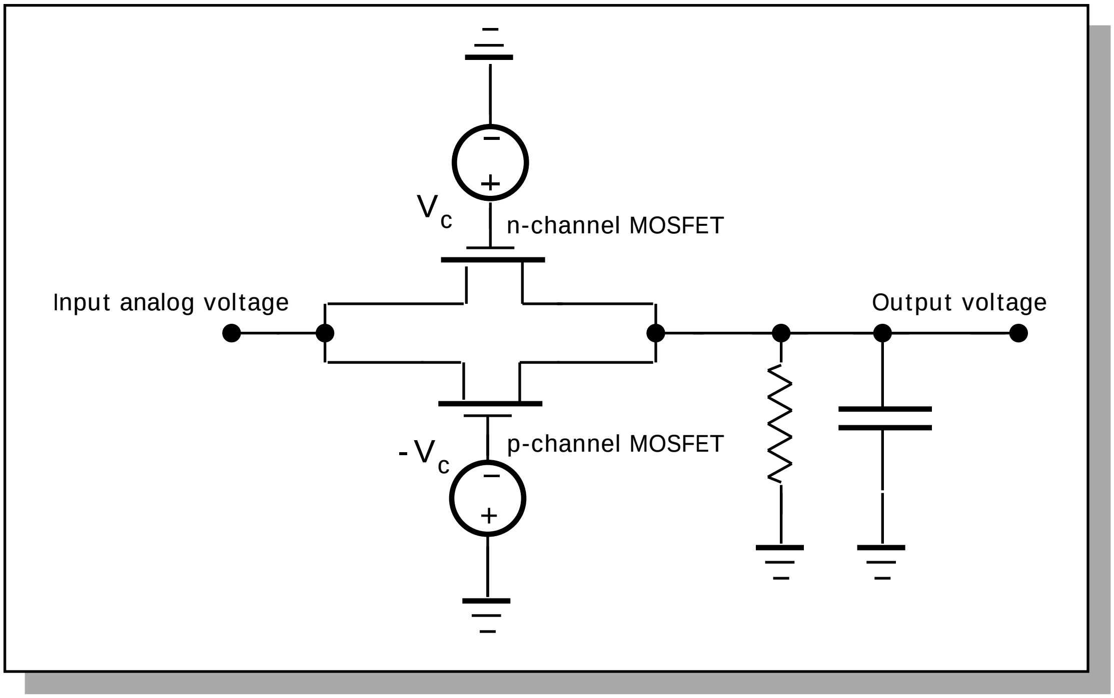
Using this characteristic, find the **common emitter current gain** (CECG) and the **common base current gain** (CBCG) of the transistor when it is operated in the "active mode." Also find the Early voltage (V_A) of the transistor.

$$\beta = \frac{(16.0 - 10.05)\ mA}{50\ \mu A} = \frac{5.95}{50} \times 10^3 \approx 120$$

$$\alpha = \frac{\beta}{\beta + 1} = \frac{120}{121} = 0.992$$

$$V_A = \frac{I(0)}{[I(V_m) - I(0)]} V_m = \frac{15.5}{[16.5 - 15.5]} 20\ V \approx 310\ V$$

c. What does the circuit illustrated below **do**? Explain how it does it..



See discussion at:
www.deas.harvard.edu/courses/es154/lectures/lecture_4/mosfet/mos_circuits/mos_circuits.html#trans_gate

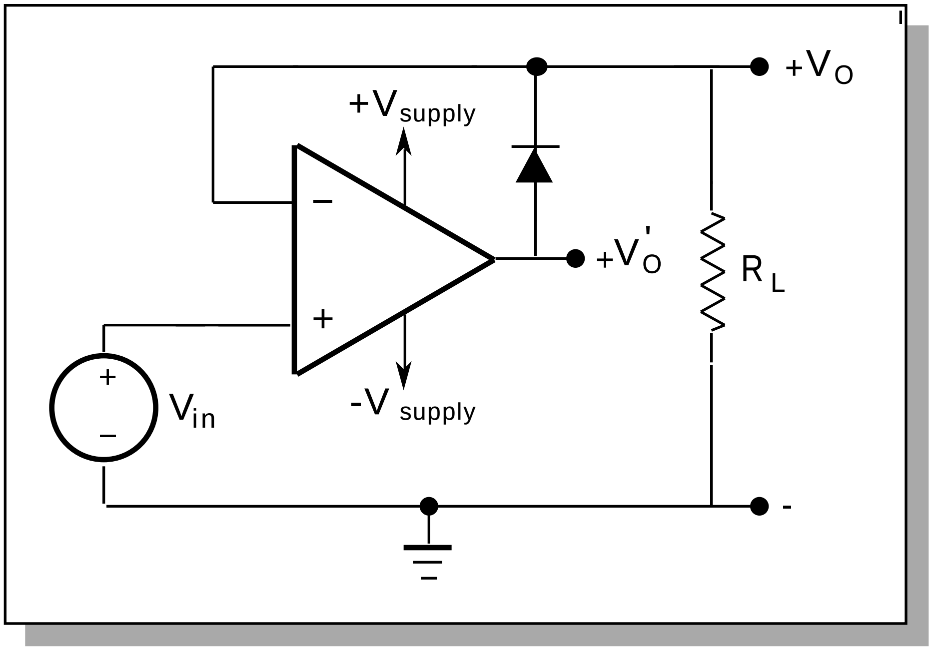
d. A **two-part** question about operational amplifier "offsets"

i.) What is meant by the "input offset voltage" of an op amp? How is it measured?

ii.) What is meant by the "input offset current" of an op amp?

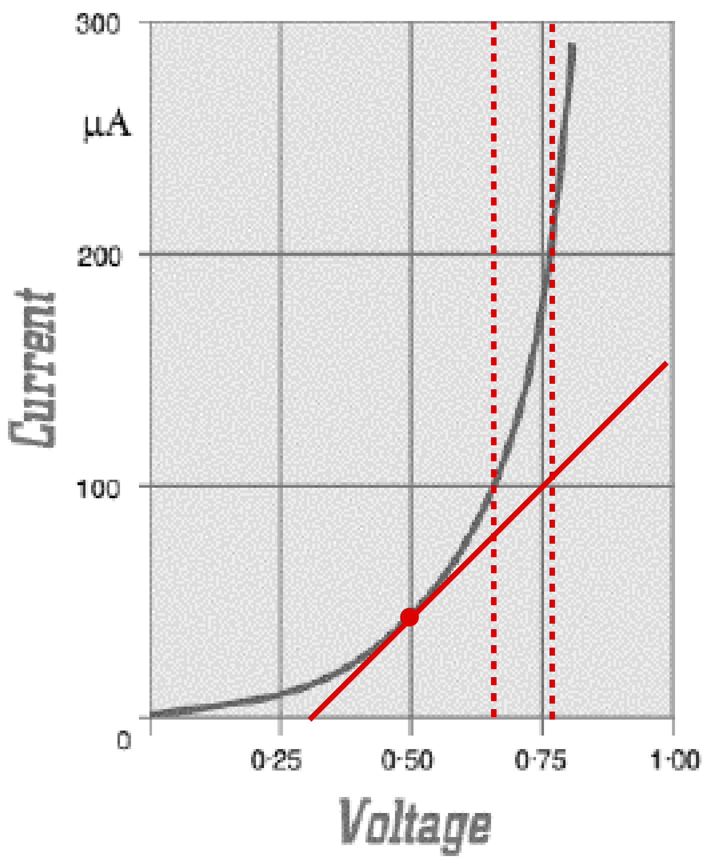
See Section 2.9 Sedra & Smith and Laboratory Assignment 1.

e. The following circuit has been described as an "improved rectifier." Explain.



See discussion at:
www.deas.harvard.edu/courses/es154/lectures/lecture_2/diode_circuits/diode_appl.html#rectifier

f. For a diode with the characteristic depicted below, calculate the effective or small-signal resistance at a forward bias of 0.5 volts.



Slope Method:

$$r_{eff} = \frac{\Delta V}{\Delta I} = \frac{0.7\,V}{155\,\mu A} = 4.5\,k\Omega$$

Analytic Method:

Assume that $I(V_1) = I_0 \left[\exp\frac{V}{V_T} - 1\right] \approx I_0 \exp\frac{V}{V_T}$

therefore $\frac{1}{r_{eff}} = \frac{d}{dV}\left[I_0 \exp\frac{V}{V_T}\right] = \frac{I(V)}{V_T}$ where $V_T = \frac{V_1 - V_2}{\ln\frac{I(V_1)}{I(V_2)}}$

From graph: $I(0.50\,V) = 45.5\,\mu A$; $I(0.66\,V) = 100\,\mu A$; $I(0.76\,V) = 200\,\mu A$

$$V_T = \frac{V_1 - V_2}{\ln\frac{I(V_1)}{I(V_2)}} = \frac{0.76 - 0.66}{\ln\frac{200}{100}} = \frac{0.10}{.693} = 0.144\,V$$

$$\frac{1}{r_{eff}} = \frac{I(V)}{V_T} = \frac{45.5\,\mu A}{144\,mV} = \frac{1}{3.2\,k\Omega}$$

g. In the space below, sketch a **complete** small signal equivalent circuit of a MOS transistor (assume that the **body** is not connected to the **source**). Identify each element of the equivalent circuit and give a "ball park" estimate of its magnitude.

See discussion at:
www.deas.harvard.edu/courses/es154/lectures/lecture_4/mosfet/mos_models/mos_models.html#body_effect

h. In the space below, draw a **cascode amplifier** stage and briefly describe the advantages this configuration offers in circuit design.

See discussion at:
www.deas.harvard.edu/courses/es154/lectures/lecture_5/lecture_5.html#cascode_amp

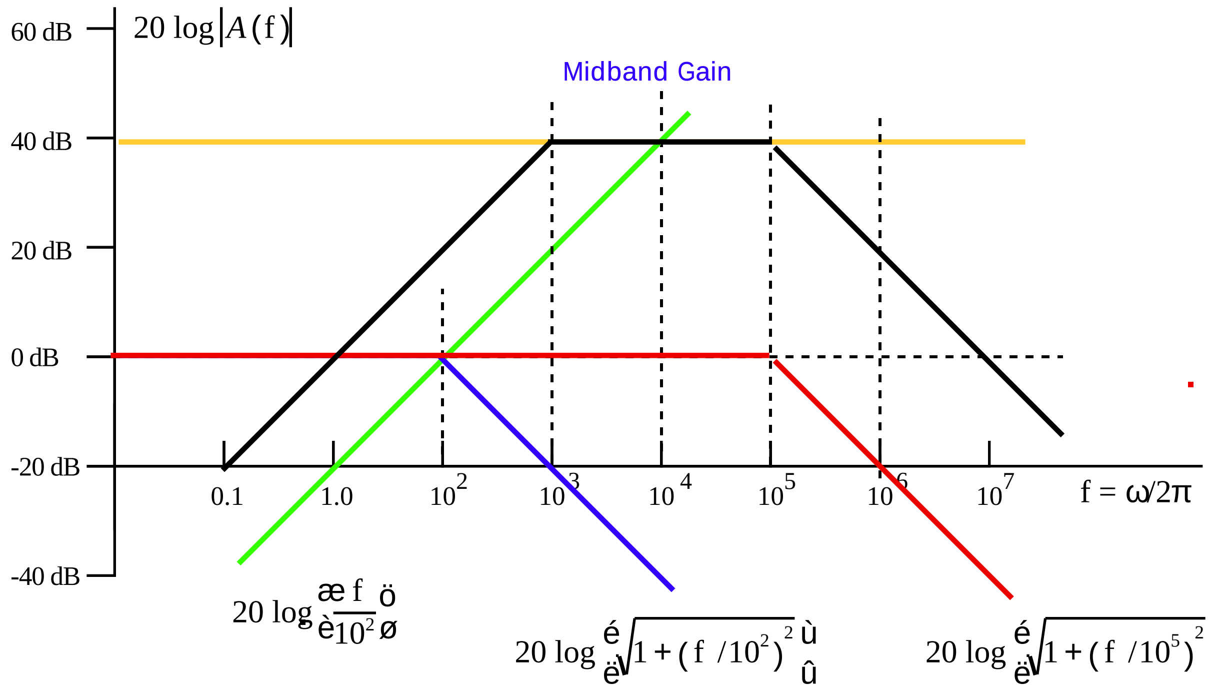
i. An amplifier has the gain transfer function

$$A(s) = 10^2 \left[\frac{s}{s + 2\pi \times 10^2}\right] \left[\frac{1}{1 + s/2\pi \times 10^5}\right]$$

In the space below sketch a Bode plot for its **magnitude** and specify the midband gain, the lower 3-dB frequency and the upper 3-dB frequency.

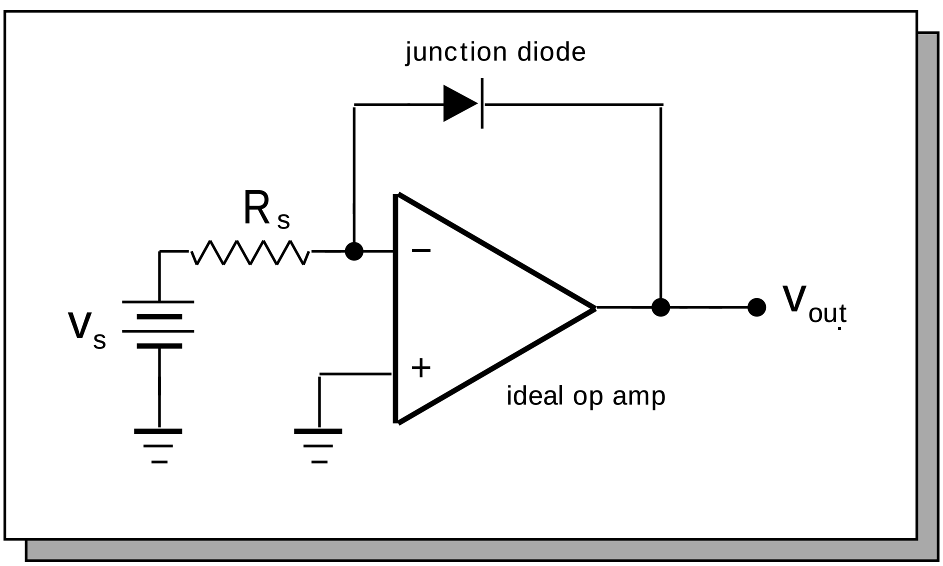
$$|A(\omega)| = 10^2 \left[\frac{\omega/2\pi \times 10^2}{\sqrt{1 + \left(\omega/2\pi \times 10^2\right)^2}}\right] \left[\frac{1}{\sqrt{1 + \left(\omega/2\pi \times 10^5\right)^2}}\right]$$

$$20 \log|A(f)| = 40 + 20 \log\left(\frac{f}{10^2}\right) - 20 \log\left[\sqrt{1 + \left(f/10^2\right)^2}\right] - 20\log\left[\sqrt{1 + \left(f/10^5\right)^2}\right]$$



See Section 7.1 and Example 7.1 in Sedra and Smith

j. The following circuit is used as a **temperature measuring device**. Find an expression for v_{out} as a function of the temperature to be measured.

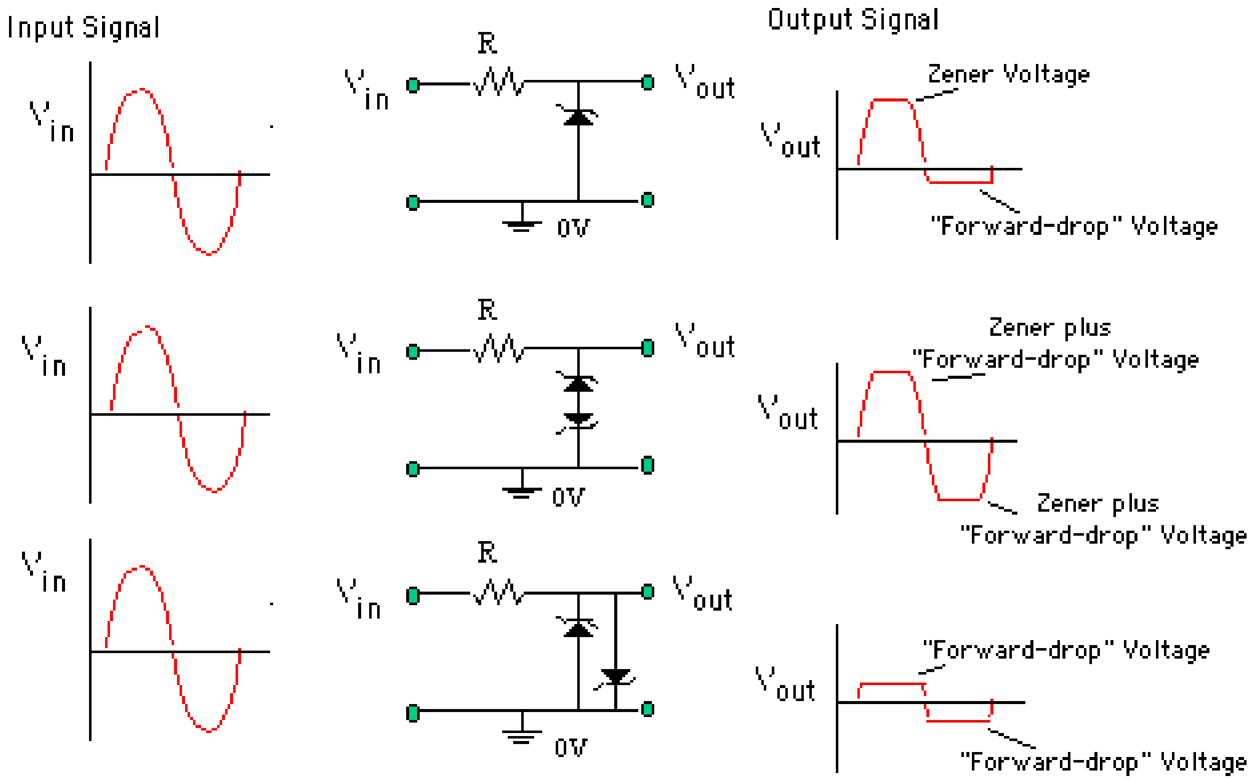


Assume that $I(V) = I_0 \left[\exp \frac{V}{n\,k\,T} - 1 \right]$

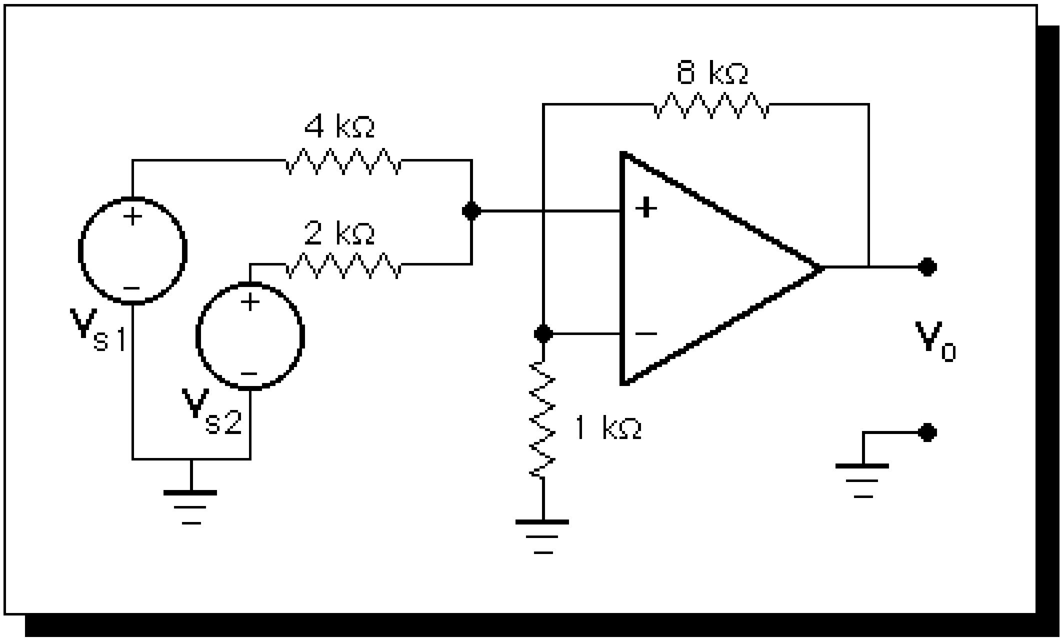
therefore $\frac{V_S}{R_S} = I_0 \left[\exp \frac{V_{OUT}}{n\,k\,T} - 1 \right]$

so that $T = \dfrac{V_{OUT}}{n\,k \ln \left(\frac{V_S}{I_0\,R_S} + 1 \right)}$

k. Consider the three Zener diode circuits illustrated below. In the spaces provide, sketch a representation of the time dependent output signal for each of the three cases



l. In the following circuit, find v_o in terms of v_{s1} and v_{s2} using the ideal op amp model.



$$V_+ = V_{S2} + 2\,k\Omega \frac{V_{S1} - V_{S2}}{6\,k\Omega} = V_{S1} - 4\,k\Omega \frac{V_{S1} - V_{S2}}{6\,k\Omega} = \frac{V_{S1} + 2\,V_{S2}}{3}$$

$$V_- = \frac{1}{9} V_{OUT}$$

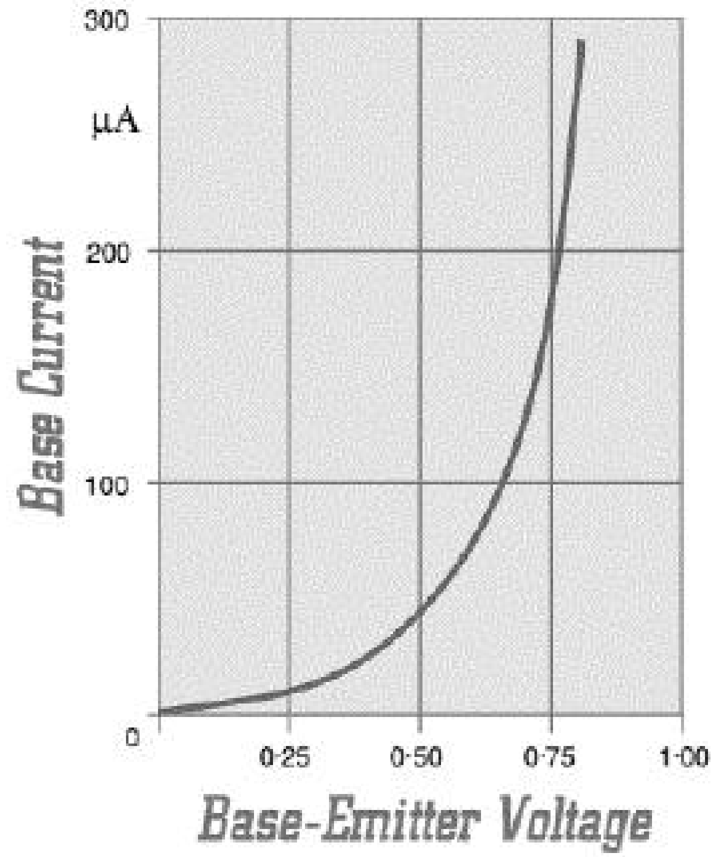
therefore $V_{OUT} = 3\left(V_{S1} + 2\,V_{S2}\right)$

m. Using expressions for the i_D-v_{DS} characteristics of an enhancement mode, n-channel MOSFET (as derived in the text and lecture), derive expressions for the small-signal **transconductance** g_m in both the triode **and** saturation regions of operation.

See discussion at:

www.deas.harvard.edu/courses/es154/lectures/lecture_4/mosfet/mos_models/mos_models.html

n. The following important characteristic curves for a particular BJT which tells us a good deal about that device performance.



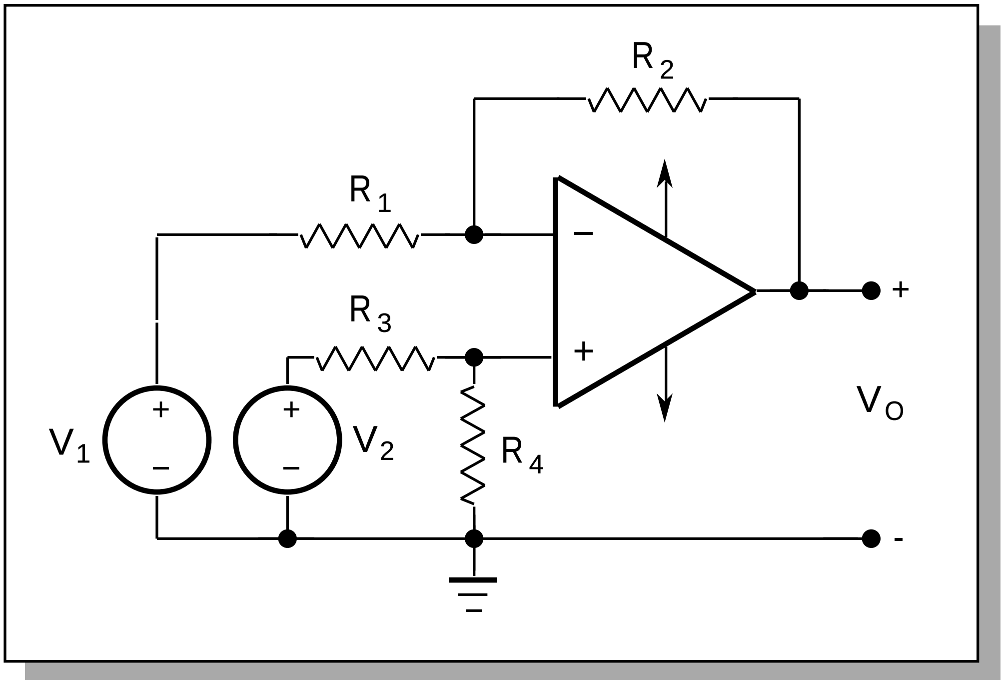
Briefly discuss the physics of this curve. What is the origin of this current? Why does have the shape that it does? What does it tell us about the given transistor's performance?

See discussion at:

www.deas.harvard.edu/courses/es154/lectures/lecture_3/collector_current/collector_current.html

2. (Prudent time allocation = 15 minutes)

For the amplifier circuit shown below, find an expression for the output v_O in terms of the two inputs v_1 and v_2. From this expression find expressions for the differential gain G_d, the common-mode gain G_{CM} and the common-mode rejection ratio CMMR.



$$\frac{v_- - v_1}{R_1} = \frac{v_0 - v_-}{R_2} \qquad v_- = \frac{R_2}{R_1 + R_2} v_1 + \frac{R_1}{R_1 + R_2} v_0$$

$$v_+ = \frac{R_4}{R_3 + R_4} v_2$$

$$v_+ = v_- \qquad \frac{R_4}{R_3 + R_4} v_2 = \frac{R_2}{R_1 + R_2} v_1 + \frac{R_1}{R_1 + R_2} v_0$$

$$v_0 = \frac{R_1 + R_2}{R_1}\left[\frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1 + R_2} v_1\right]$$

$$= \frac{R_1 + R_2}{R_1}\left[\frac{R_4}{R_3 + R_4}\left(v_{cm} + \frac{v_d}{2}\right) - \frac{R_2}{R_1 + R_2}\left(v_{cm} - \frac{v_d}{2}\right)\right]$$

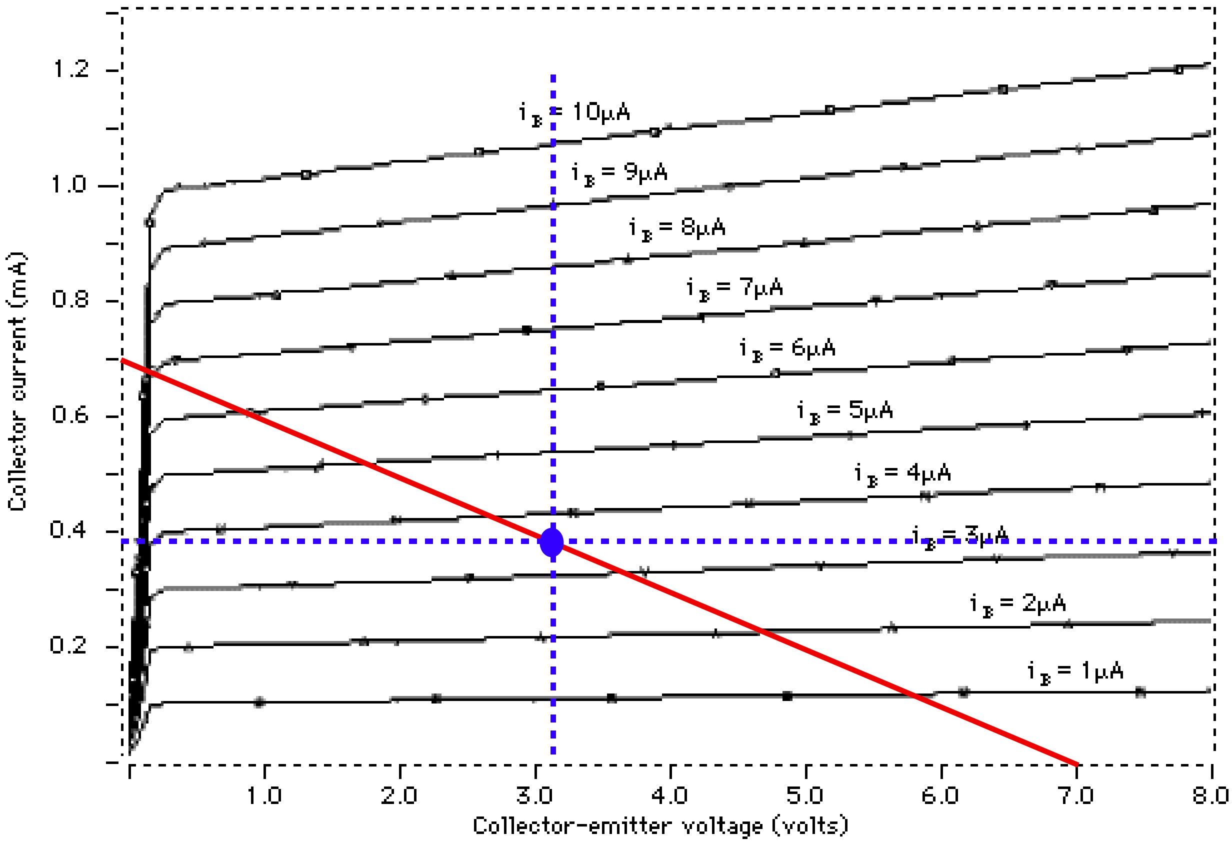
$$= \frac{v_d}{2}\left[\frac{R_2}{R_1} + \frac{R_4\left(R_1 + R_2\right)}{R_1\left(R_3 + R_4\right)}\right] + v_{cm}\left[\frac{R_4 R_1 - R_2 R_3}{R_1\left(R_3 + R_4\right)}\right]$$

$$G_d = \frac{1}{2}\left[\frac{R_2}{R_1} + \frac{R_4\left(R_1 + R_2\right)}{R_1\left(R_3 + R_4\right)}\right] \qquad G_{cm} = \frac{R_4 R_1 - R_2 R_3}{R_1\left(R_3 + R_4\right)}$$

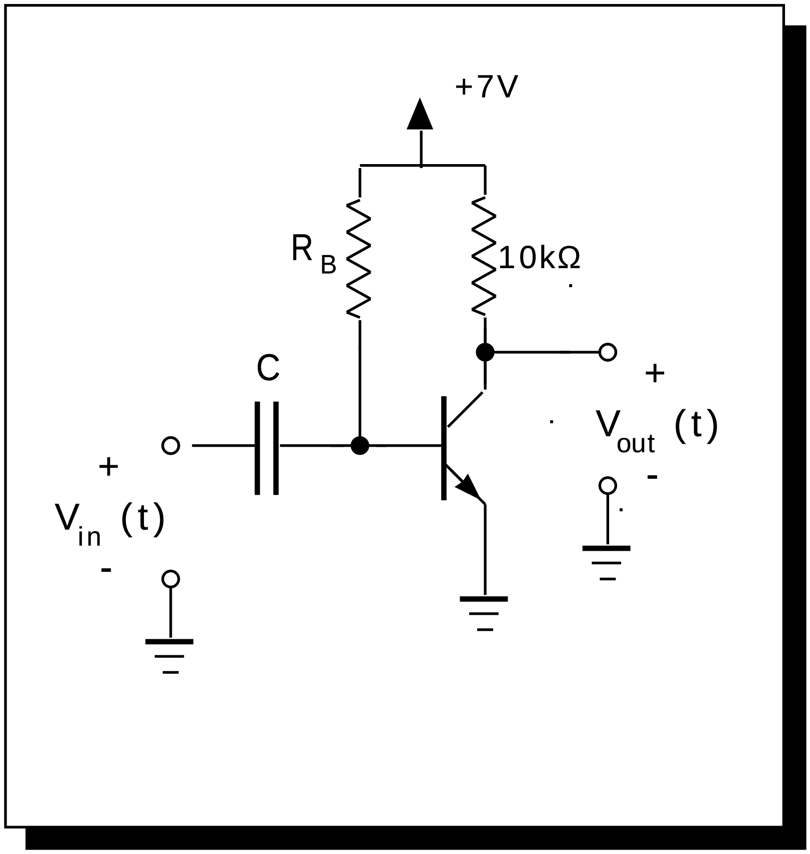
$$CMRR = 20 \log\left|\frac{G_d}{G_{cm}}\right| = 20 \log\left|\frac{1}{2}\,\frac{R_2\left(R_3 + R_4\right) + R_4\left(R_1 + R_2\right)}{R_4 R_1 - R_2 R_3}\right|$$

3. (Prudent time allocation = 30 minutes)

Consider an npn BJT with the following I_C-V_{CE} characteristic:



Suppose that such a transistor is used in the circuit illustrated below.



a. By drawing a **load line** on the characteristic curve, choose the circuit **quiescent point** or DC operating point to maximize the AC voltage swing of $V_{out}(t)$. What are the DC values of the bias current, the collector current, R_B and V_{out} at the quiescent point (carefully specify units)?

DC bias current = 3.5 μA

DC collector current = 0.39 mA

DC output voltage = 3.2 V

Bias resistor = (7.0 – 0.7)/3.5 x 10^{-6} = 1.8 MΩ

b. For these quiescent point values, sketch in the space on the next page a **complete small-signal equivalent circuit** of the transistor including values and units for all parameters of the equivalent circuit (neglect any high frequency effects).

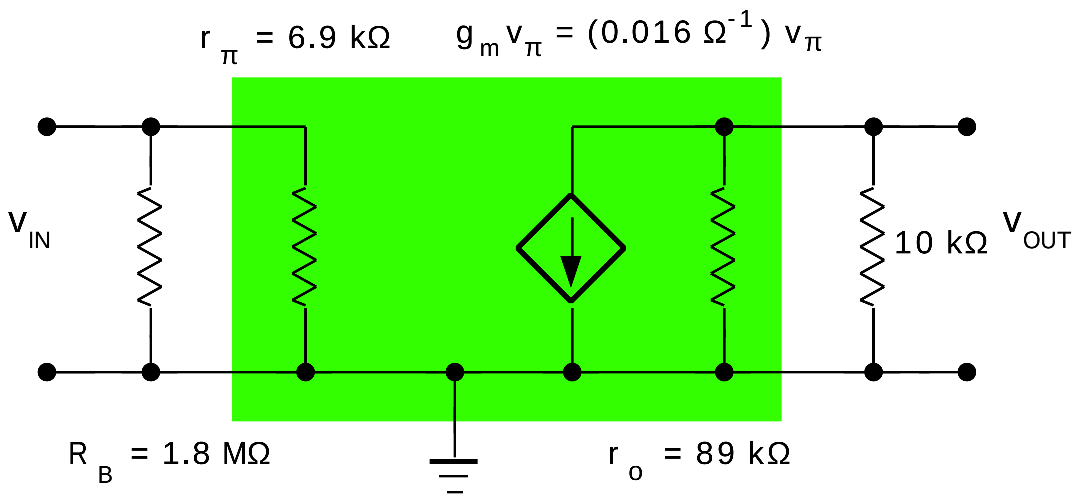
From the graph at the Q - point:

$$\beta = \frac{0.33\ mA}{3\ \mu A} \approx 110$$

$$g_m = \frac{I_c}{V_T} = \frac{0.39\ mA}{25\ mV} \approx 0.016\ \Omega^{-1}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{110}{0.016\ \Omega^{-1}} \approx 6.9\ k\Omega$$

$$r_o = \frac{\Delta v_{ce}}{\Delta i_c} = \frac{8\ V}{0.09\ mA} = 89\ k\Omega$$



c. Again at the quiescent point found above and at frequencies where we can neglect capacitive effects, find the small-signal voltage gain, input impedance and output impedance of the circuit.

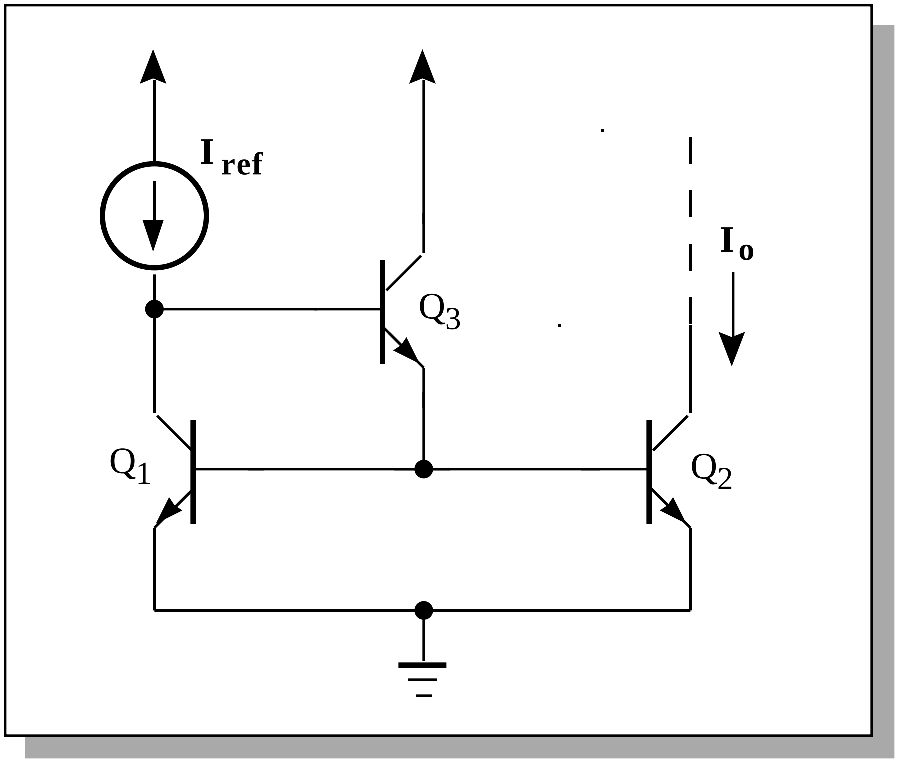
Small-signal voltage gain = $(0.016\ \Omega^{-1})$ (89 || 10) kΩ = 144

Small-signal input impedance = 1.8 MΩ || 6.9 kΩ = 6.9 kΩ

Small-signal output impedance = (89 || 10) kΩ =9.0 kΩ

4. (Prudent time allocation = 15 minutes)

If in the following circuit we assume that the three transistors are identical, find an expression for the ratio $\mathbf{I_o/I_{ref}}$ in terms of the β of the transistors.

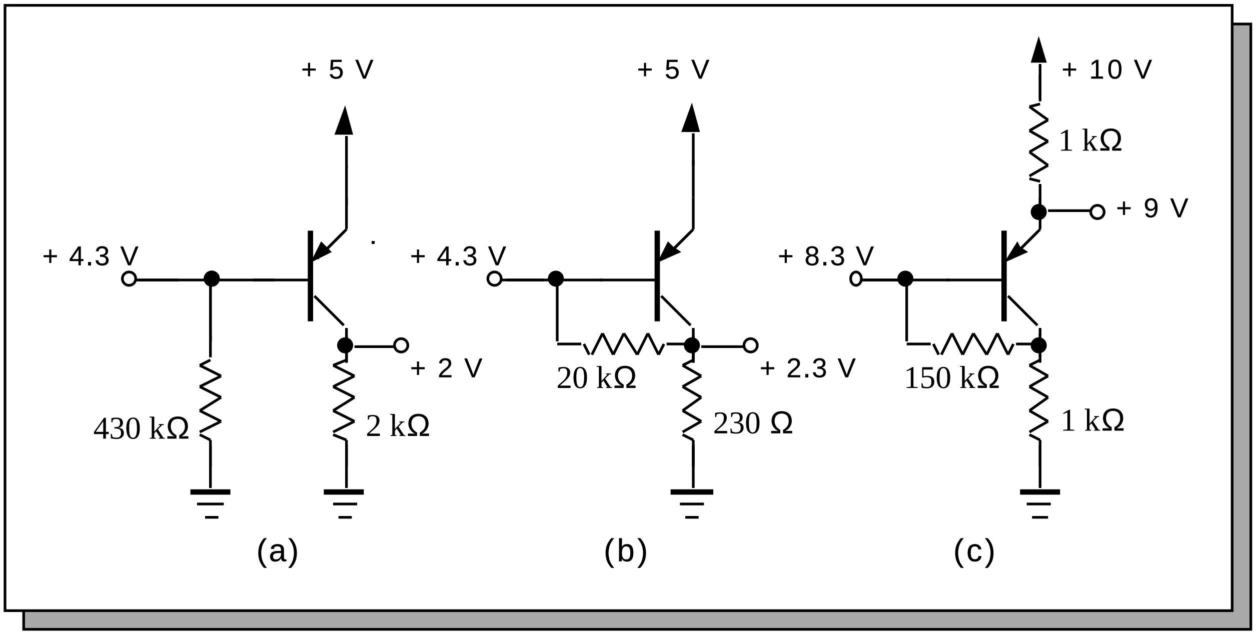


See the discussion at:

www.deas.harvard.edu/courses/es154/lectures/lecture_6/mirrors/mirrors.html#base_comp

5. (Prudent time allocation = 10 minutes)

Measurements on the three circuits below yield the voltages indicated. Find the value of β for each of the pnp transistors.



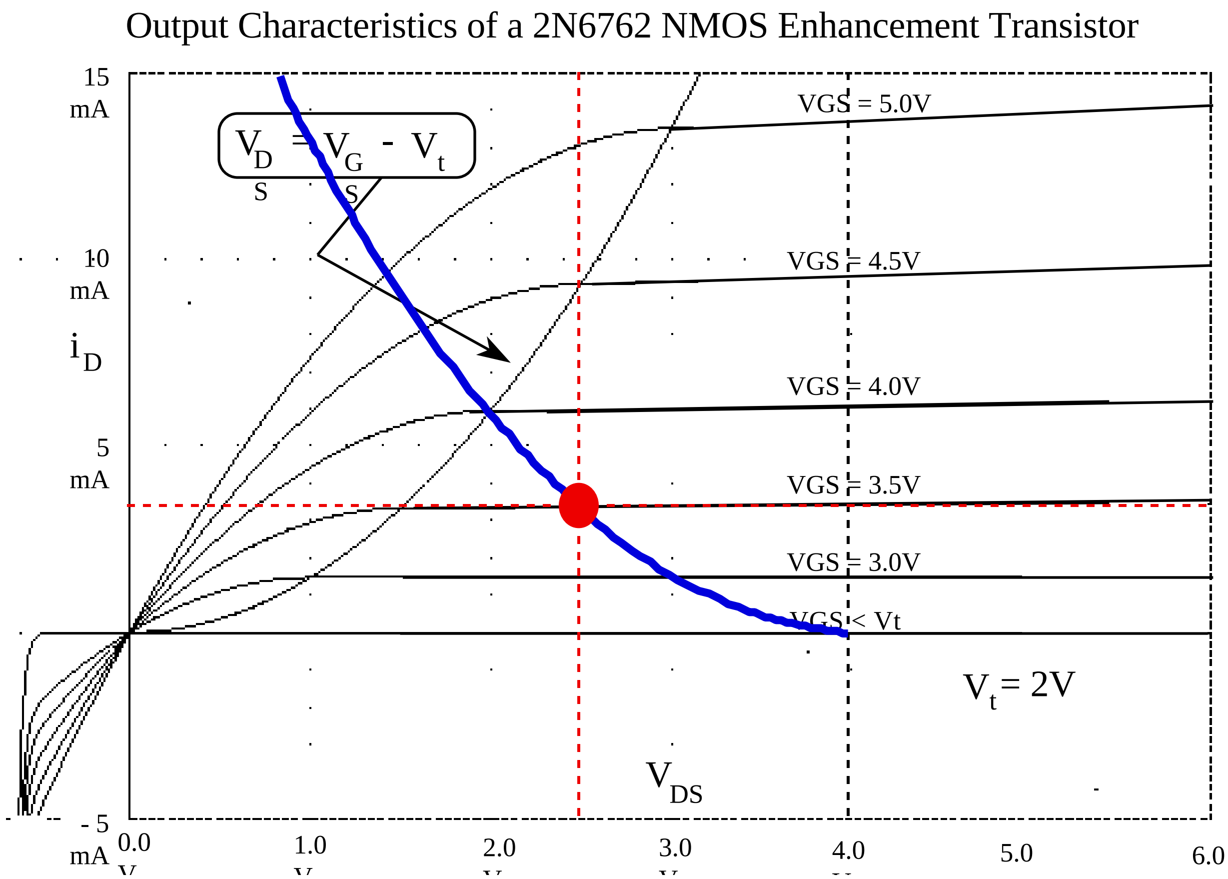
(a) β =

(b) β =

(c) β =

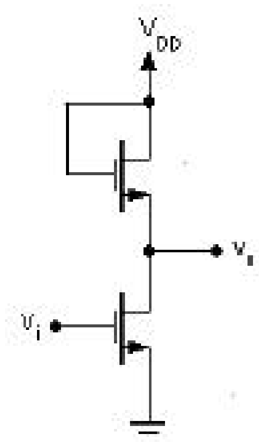
6. (Prudent time allocation = 20 minutes)

Consider a NMOS enhancement transistor with the following characteristics:



Suppose two such transistors are used in a common source, NMOS amplifier configuration where one transistor serves as the load of the other. Assume that the amplifier is powered by a single-sided + 6 volt supply.

a. Draw a circuit of such an enhancement loaded amplifier in the space below:



b. Draw directly on the characteristic curve above the appropriate **load curve** for the amplifier. **See characteristic curve**

c. Using this load curve, choose a **quiescent point** or **dc operating point** so as to maximize the ac voltage swing of the amplifier output. What are the dc values of the bias voltage, the drain current, and the drain-source voltage at the quiescent point (carefully specify units)?

dc bias voltage = 3.5 V

dc drain current = 3 mA

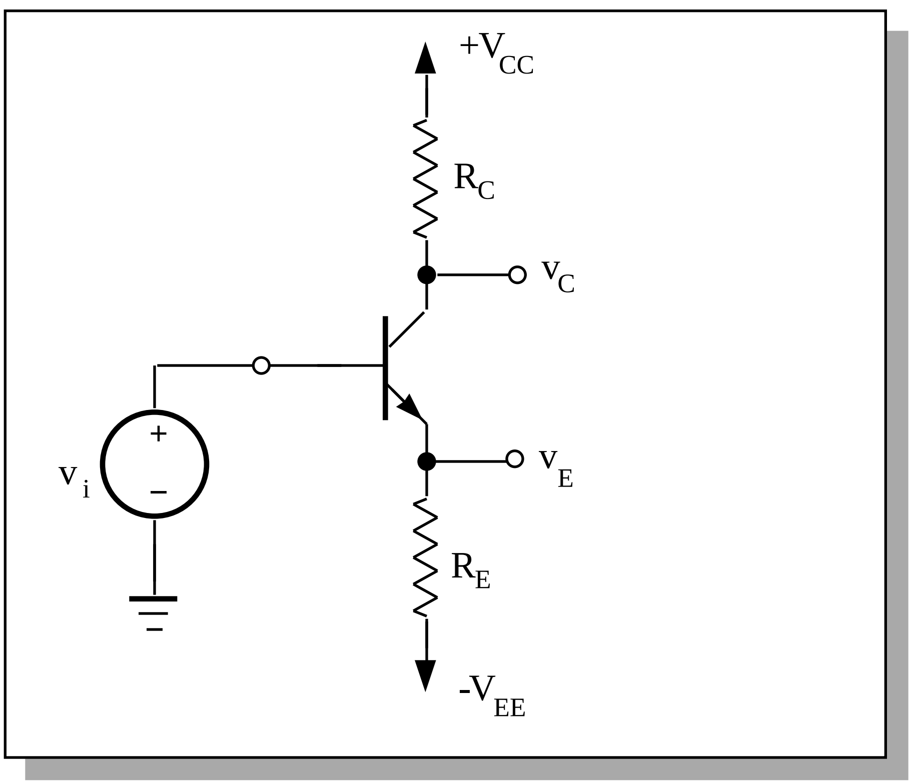
dc drain-source voltage = 2.5 V

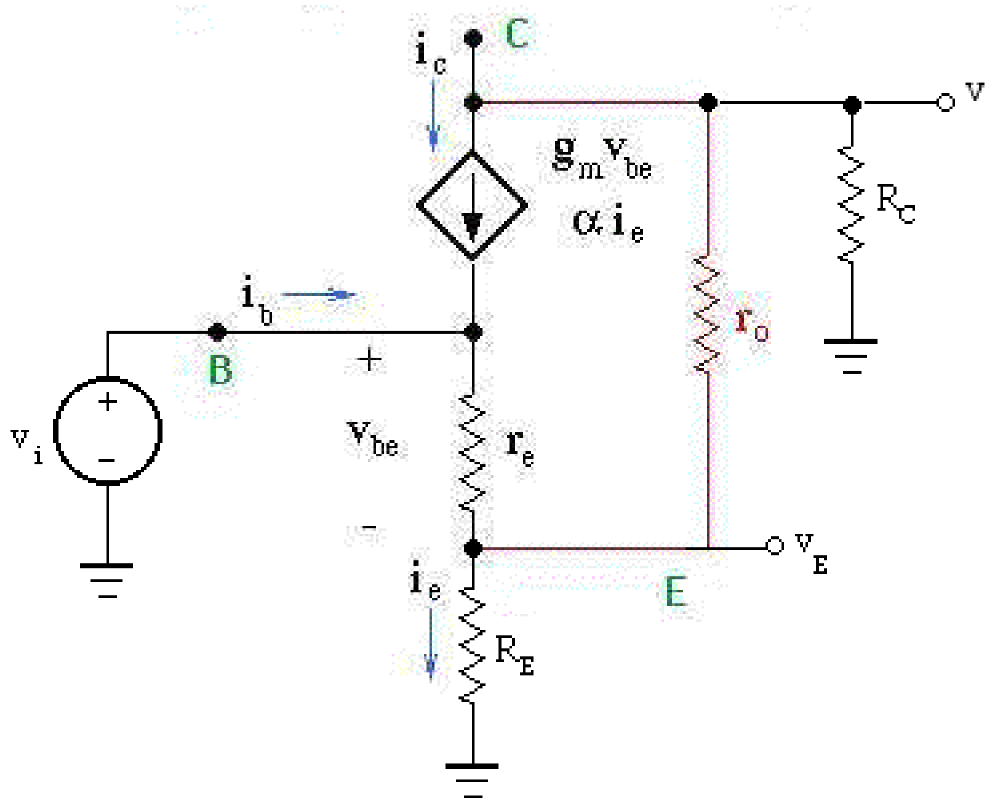
d. For these quiescent point values, find the ac voltage gain of the amplifier.

voltage gain ≈ 1

7. (Prudent time allocation = 15 minutes)

As the first step in analyzing the following BJT amplifier, replace the transistor with its low-frequency "T" equivalent circuit. Then, derive the gain v_E/v_i, the gain v_C/v_i, and the input impedance of the amplifier.





Nodal Analysis (neglecting r_o - *i.e* $r_o \quad \infty$):

Node B

$$\mathrm{i_b} + g_m\left(\mathrm{v_i} - \mathrm{v_E}\right) + \frac{\mathrm{v_E}}{r_e} = \frac{\mathrm{v_i}}{r_e}$$

Node C

$$g_m\left(\mathrm{v_i} - \mathrm{v_E}\right) + \frac{\mathrm{v_C}}{R_C} = 0$$

Node E

$$\frac{\mathrm{v_E}}{r_e \parallel R_E} = \frac{\mathrm{v_i}}{r_e}$$

Nodal Analysis (including r_o):

Node B

$$\mathrm{i_b} + g_m\left(\mathrm{v_i} - \mathrm{v_E}\right) + \frac{\mathrm{v_E}}{r_e} = \frac{\mathrm{v_i}}{r_e}$$

Node C

$$g_m\left(\mathrm{v_i} - \mathrm{v_E}\right) + \frac{\mathrm{v_C}}{r_o \parallel R_C} = \frac{\mathrm{v_E}}{r_o}$$

Node E

$$\frac{\mathrm{v_E}}{r_o} + \frac{\mathrm{v_E}}{r_e \parallel R_E} = \frac{\mathrm{v_C}}{r_o} + \frac{\mathrm{v_i}}{r_e}$$

Results (neglecting r_o - *i.e* $r_o \quad \infty$):

$$\frac{\mathrm{v_E}}{\mathrm{v_i}} = \frac{r_e \parallel R_E}{r_e} = \frac{R_E}{r_e + R_E}$$

$$\frac{\mathrm{v_C}}{\mathrm{v_i}} = -g_m R_C \left(\frac{r_e}{r_e + R_E}\right)$$

$$\frac{1}{R_{in}} = \frac{\mathrm{i_b}}{\mathrm{v_i}} = -\left(\frac{r_e}{r_e + R_E}\right)\left(g_m - \frac{1}{r_e}\right)$$

Results (including r_o):

$$\frac{v_E}{v_i} = \frac{\dfrac{r_e \parallel R_E}{r_e} - g_m \left(\dfrac{r_o \parallel R_C}{r_o}\right)\left(r_e \parallel R_E\right)}{1 - g_m \left(\dfrac{r_o \parallel R_C}{r_o}\right)\left(r_e \parallel R_E\right) + \left(\dfrac{r_o \parallel R_C}{r_o\, R_C}\right)\left(r_e \parallel R_E\right)}$$

$$\frac{v_C}{v_i} = \frac{-g_m \left(r_o \parallel R_C\right)\left(\dfrac{r_e \parallel R_E}{r_o}\right)\left(1 + \dfrac{r_o}{R_E}\right) + \left(\dfrac{r_e \parallel R_E}{r_e}\right)\left(\dfrac{r_o \parallel R_C}{r_o}\right)}{1 - g_m \left(\dfrac{r_o \parallel R_C}{r_o}\right)\left(r_e \parallel R_E\right) + \left(\dfrac{r_o \parallel R_C}{r_o\, R_C}\right)\left(r_e \parallel R_E\right)}$$ s

$$\frac{1}{R_{in}} = \frac{i_b}{v_i} = -\left[\frac{\dfrac{r_e \parallel R_E}{R_E} + \left(\dfrac{r_o \parallel R_C}{r_o\, R_C}\right)\left(r_e \parallel R_E\right)}{1 - g_m \left(\dfrac{r_o \parallel R_C}{r_o}\right)\left(r_e \parallel R_E\right) + \left(\dfrac{r_o \parallel R_C}{r_o\, R_C}\right)\left(r_e \parallel R_E\right)}\right]\left(g_m - \frac{1}{r_e}\right)$$

8. (Prudent time allocation = 10 minutes)

For a particular npn transistor operating in the active mode the collector current is measured to be 1 mA and 10 mA for base-to-emitter voltages of 0.63 V and 0.70 V, respectively. Find the corresponding values of n and I_S for this transistor.

Assume that $I_C\left(V_{BE}\right) = I_S \exp \dfrac{V_{BE}}{n V_T}$

therefore $$n = \left(\frac{1}{V_T}\right)\frac{\left(V_1 - V_2\right)}{\ln \dfrac{I\left(V_1\right)}{I\left(V_2\right)}} = \left(\frac{1}{25\ mV}\right)\frac{\left(0.7 - 0.63\right) V}{\ln \dfrac{10}{1}} = \left(\frac{70\ mV}{25\ mV}\right)\left(\frac{1}{2.3}\right) = 1.2$$

and $$I_S = \frac{I_C\left(V_{BE}\right)}{\exp \dfrac{V_{BE}}{n\, V_T}} = \frac{10\ mA}{\exp \dfrac{700\ mV}{30\ mV}} = \frac{10\ mA}{1.36 \times 10^{10}} = 0.74 \times 10^{-12}\ \text{A}$$

a. If two such devices are connected in parallel and a forward bias o 0.65 V is applied across the two base-emitter junctions, what total collector current do you expect?

$$\sum I_C\left(650\ mV\right) = 2 \times \left(0.74 \times 10^{-12}\ \text{A}\right) \exp \frac{650\ mV}{30\ mV} = 3.8\ mA$$

9. (Prudent time allocation = 12 minutes

a. Draw a complete circuit diagram of an emitter-follower amplifier which uses an npn BJT.

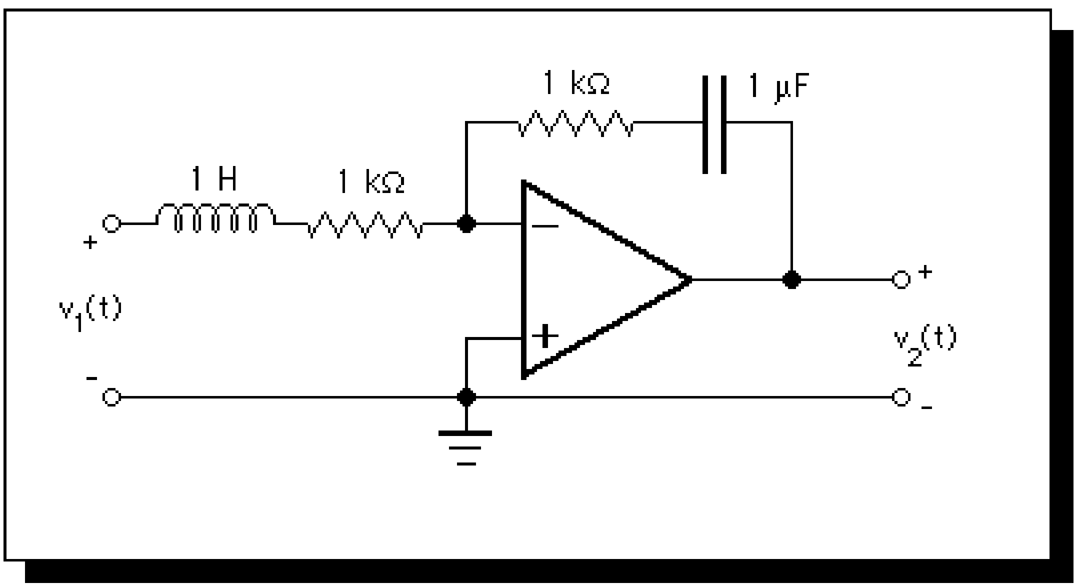
b. Draw a small-signal version of this complete emitter-follower that utilizes the most appropriate BJT small-signal equivalent circuit.

c. Using this small-signal circuit, find an expression for the voltage gain of the amplifier

d. Again using this small-signal circuit, find an expression for the input impedance of the amplifier.

See discussion at:
www.deas.harvard.edu/courses/es154/lectures/lecture_3/bjt_amps/bjt_amps.html#ce_amp

10. (Prudent time allocation = 10 minutes)



a. Assuming that the op amp is *ideal*, find the **transfer function** $\mathbf{H}(s) = \mathbf{V}_2(s)/\mathbf{V}_1(s)$.

$$\frac{V_1(s)}{10^3 + s} = \frac{-V_2(s)}{10^3 + \dfrac{10^6}{s}}$$

$$H(s) = \frac{V_2(s)}{V_1(s)} = -\frac{10^3 + \dfrac{10^6}{s}}{10^3 + s} = -\frac{1 + \dfrac{10^3}{s}}{1 + \dfrac{s}{10^3}}$$

b. Describe the behavior of this transfer function in both the high and low frequency limits.

$$H(s) = \frac{V_2(s)}{V_1(s)} \xrightarrow[\omega \to \infty]{} -\frac{10^3}{s} \quad 0$$

$$H(s) = \frac{V_2(s)}{V_1(s)} \xrightarrow[\omega \to 0]{} -\frac{10^3}{s} \quad \infty$$

11. (Prudent time allocation = 15 minutes)

a. Draw a complete circuit diagram of an source-follower amplifier which uses an n-channel MOSFET.

b. Draw a small-signal version of this complete source-follower that utilizes the most appropriate MOSFET small-signal equivalent circuit.

c. Using this small-signal circuit, find an expression for the voltage gain of the amplifier

c. Again using this small-signal circuit, find an expression for the input impedance of the amplifier.

See and adapt the "The Common-Collector Amplifier or Emitter Follower" discussion on page 290-295 in Sedra & Smith.