## Two-Port Parameters for Single-Stage Amplifiers

<table>
<thead>
<tr>
<th>Amplifier Type</th>
<th>Controlled Source</th>
<th>Input Resistance $R_{in}$</th>
<th>Output Resistance $R_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Common Emitter</td>
<td>$G_m = g_m$</td>
<td>$r_\pi$</td>
<td>$r_o \parallel r_{oc}$</td>
</tr>
<tr>
<td>Common Source</td>
<td>$G_m = g_m$</td>
<td>infinity</td>
<td>$r_o \parallel r_{oc}$</td>
</tr>
<tr>
<td>Common Base</td>
<td>$A_i = -1$</td>
<td>$1 / g_m$</td>
<td>$r_{oc} \parallel [(1 + g_m(r_\pi</td>
</tr>
<tr>
<td>Common Gate</td>
<td>$A_i = -1$</td>
<td>$1 / g_m$, ($v_{sb} = 0$) -otherwise- $1 / (g_m + g_{mb})$</td>
<td>$r_{oc} \parallel [(1 + g_m R_S)r_o]$, ($v_{sb}=0$) -otherwise- $r_{oc} \parallel [(1 + (g_m + g_{mb})R_S)r_o]$, both for $r_o &gt;&gt; R_S$</td>
</tr>
<tr>
<td>Common Collector</td>
<td>$A_v = 1$</td>
<td>$r_\pi + \beta_o(r_o \parallel r_{oc} \parallel R_L)$</td>
<td>$(1 / g_m) + R_S / \beta_o$</td>
</tr>
<tr>
<td>Common Drain</td>
<td>$A_v = 1$ if $v_{sb} = 0$, -otherwise- $g_m / (g_m + g_{mb})$</td>
<td>infinity</td>
<td>$1 / g_m$ if $v_{sb} = 0$, -otherwise- $1 / (g_m + g_{mb})$</td>
</tr>
</tbody>
</table>

Note: appropriate two-port model is used, depending on controlled source
Sinusoidal Function Review

Sinusoidal functions are important in analog signal processing.

\[ v(t) = v \cos(\omega t + \phi) \]

- **amplitude**: (half of peak-to-peak)
- **frequency**: (radian) \( \omega = 2\pi f = 2\pi (1/T) \)
- **phase**: (degrees or radians)

\[ v_1(t) = v \cos(\omega t) \]
\[ v_2(t) = v \cos(\omega t - 45) \]
\[ \omega = \frac{2\pi}{T} \]

1. EECS 20/120: periodic functions can be represented as sums of sinusoids functions at different frequencies.

2. The response of a circuit to a sinusoidal input signal, as a function of the frequency, leads to insights into the behavior of the circuit.
**Frequency Response**

**Key concept**: small-signal models for amplifiers are *linear* and therefore, cosines and sines are solutions of the linear differential equations which arise from $R$, $C$, and controlled source (e.g., $G_m$) networks.

* The problem: finding the solutions to the differential equations is TEDIOUS and provides little insight into the behavior of the circuit!
Phasors

It is much more efficient to work with imaginary exponentials as “representing” the sinusoidal voltages and currents ... since these functions are solutions of linear differential equations and

$$\frac{d}{dt}(e^{j\omega t}) = j\omega(e^{j\omega t})$$

How to connect the exponential to the measured function $v(t)$? Conventionally, $v(t)$ is the real part of the imaginary exponential

$$v(t) = v \cos(\omega t + \phi) \rightarrow \text{Re}(ve^{(j\omega t + \phi)}) = \text{Re}(ve^{j\phi} e^{j\omega t})$$

where $v$ is the amplitude and $\phi$ is the phase of the sinusoidal signal $v(t)$.

The **phasor** $V$ is defined as the complex number

$$V = ve^{j\phi}$$

Therefore, the measured function is related to the phasor by

$$v(t) = \text{Re}(Ve^{j\omega t})$$
The current through a capacitor is proportional to the derivative of the voltage:

\[ i(t) = C \frac{d}{dt} v(t) \]

We assume that all signals in the circuit are represented by sinusoids. Substitution of the phasor expression for voltage leads to:

\[ v(t) \rightarrow V e^{j\omega t} \quad \ldots \quad I e^{j\omega t} = C \frac{d}{dt} (V e^{j\omega t}) = j\omega CV e^{j\omega t} \]

which implies that the ratio of the phasor voltage to the phasor current through a capacitor (the \textit{impedance}) is

\[ Z(j\omega) = \frac{V}{I} = \frac{1}{j\omega C} \]

Implication: the phasor current is \textit{linearly proportional} to the phasor voltage, making it possible to solve circuits involving capacitors and inductors as rapidly as resistive networks ... as long as all signals are sinusoidal.
Phasor Analysis of the Low-Pass Filter

* Voltage divider with impedances --

Replacing the capacitor by its impedance, $1 / (j\omega C)$, we can solve for the ratio of the phasors $V_{out} / V_{in}$

\[
\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{R + 1/j\omega C}
\]

multiplying by $j\omega C/j\omega C$ leads to

\[
\frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}
\]
Frequency Response of LPF Circuits

The phasor ratio \( V_{out} / V_{in} \) is called the transfer function for the circuit.

How to describe \( V_{out} / V_{in} \)?

- complex number ... could plot \( \text{Re}(V_{out} / V_{in}) \) and \( \text{Im}(V_{out} / V_{in}) \) versus frequency
- polar form translates better into what we measure on the oscilloscope ... the magnitude (determines the amplitude) and the phase

* "Bode plots":

- magnitude and phase of the phasor ratio: \( V_{out} / V_{in} \)
- range of frequencies is very wide (DC to \( 10^{10} \) Hz, for some amplifiers)
  - therefore, plot frequency axis on log scale
- range of magnitudes is also very wide:
  - therefore, plot magnitude on log scale

Convention: express the magnitude in decibels “dB” by

\[
\left| \frac{V_{out}}{V_{in}} \right| \ dB = 20 \log \left( \frac{V_{out}}{V_{in}} \right)
\]

phase is usually expressed in degrees (rather than radians):

\[
\angle \frac{V_{out}}{V_{in}} = \text{atan} \left( \frac{\text{Im}(V_{out}/V_{in})}{\text{Re}(V_{out}/V_{in})} \right)
\]
Complex Algebra Review

* Magnitudes:

\[
\frac{|Z_1|}{|Z_2|} = \frac{|Z_1|}{|Z_2|} = \frac{\sqrt{X_1^2 + Y_1^2}}{\sqrt{X_2^2 + Y_2^2}}, \text{ where}
\]

\[
Z_1 = X_1 + jY_1 \quad Z_2 = X_2 + Y_2
\]

* Phases:

\[
\angle \frac{Z_1}{Z_2} = \angle Z_1 - \angle Z_2 = \tan^{-1} \frac{Y_1}{X_1} - \tan^{-1} \frac{Y_2}{X_2}
\]

* Examples: