Design of a low cost Zimm–Crothers viscometer: From theory to experiment

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To accurately measure low viscosities of liquids, we describe how a Zimm–Crothers viscometer works and how to build it. The viscometer involves the action of a rotating magnetic field on a metallic cylinder floating on the liquid to be studied. The principles of electromagnetism and fluid mechanics involved make the viscometer an excellent tool for undergraduate laboratory courses and for measuring the shear viscosity of low viscous fluids. We discuss the advantages and limitations of this inexpensive and easy to use apparatus compared to other classical techniques. Calibrations with Newtonian fluids are explained and experiments with Non-Newtonian materials are discussed.

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I. INTRODUCTION

Viscosity characterizes the resistance of fluids to flow. Measuring it with accuracy is fundamental for many industrial applications because its value controls the nature of the flow (laminar or turbulent).[1] The viscosity also is of much interest in chemical physics and biological sciences. As an example, its value for dilute solutions of macromolecules allows the determination of the volume fraction of the dispersed phase and eventually the size of the solid suspension, as well as the radius of gyration of the solute.[2]

There are several methods for measuring the viscosity of a liquid. Capillary viscometry consists of forcing the liquid through a cylindrical capillary tube by applying an excess pressure, ΔP, at one of the two capillary ends. For Newtonian fluids, the resulting flow rate, Q, varies linearly with ΔP. This experimental method gives accurate results for low and moderate viscous Newtonian fluids. However, it has two important drawbacks. When the fluid is non-Newtonian, the velocity profile inside of the capillary tube is not parabolic, and the relation between Q and the applied pressure gradient $\Delta P/L$, where L is the capillary tube length, is no longer linear. Consequently, a direct determination of the rheological behavior of the fluid becomes tedious because it requires knowing the velocity profile, which in turn depends on the fluid response itself. In addition, for complex fluids such as lyotropic lamellar phases or micellar solutions (mixture of water and/or oil with surfactant molecules), strong couplings exist between the flow field and the fluid microstructure. Thus, reaching a steady state may take much longer than the residential time of the fluid in the capillary tube.[3]

To overcome this problem, we can use rotational viscometers to indefinitely shear a material. The simplest way consists of sandwiching the fluid between two concentric cylinders of respective radii r_1 and $r_2 > r_1$, where one of the two cylinders is fixed while the other is free to rotate. In stress-controlled experiments, the torque, T, is imposed on the free cylinder and the resulting angular velocity, ω, is measured. For a gap spacing $e = r_2 - r_1$ smaller than about 10% of r_1, both the stress, σ, and the shear rate, $\dot{\gamma}$, are nearly uniform, so that the fluid viscosity, $\eta = \sigma/\dot{\gamma}$, and more generally, its rheological response, $\sigma(\dot{\gamma})$, can be straightforwardly determined from the experimental relation between T and ω. However, this type of commercial apparatus is expensive, and despite the use of air bearings, the measurement of low fluid viscosities remains difficult because of frictional forces that always exist between the rotation axis and other mechanical devices.

To avoid this inherent difficulty of rotational viscometry, Zimm and Crothers came up with an original design.[4] The essence of their design consists of using a freely rotating inner cylinder, which is supported by its own buoyancy and placed in a horizontal rotating applied magnetic field. If a metallic ring is inserted inside the inner cylinder, a magnetic torque results, leading to the rotation of this cylinder. Because the moving cylinder is only in contact with the fluid, the sole frictional force results from the fluid itself. This feature enables the measurement of low viscosities with very good accuracy. In the following, we describe how this apparatus works and how to construct it.

II. THEORY

A. Electromagnetic action

Let us consider an inner vertical glass cylinder (axis Oz) whose radius and length are respectively denoted by r_1 and $\ell \gg r_1$. This cylinder is placed in a horizontal magnetic field of magnitude B_0,

$$\vec{B} = B_0[\cos(\omega t)\vec{u}_x + \sin(\omega t)\vec{u}_y], \tag{1}$$

rotating along the vertical axis with angular frequency ω. In Eq. (1), $\vec{u}_x$ and $\vec{u}_y$ are orthogonal unit vectors in the horizontal plane corresponding to the direction of the axis of the coils. We place a metallic aluminum ring whose height, radius, and electrical conductivity are, respectively ℓ_0, a_0, and γ, inside the vertical glass cylinder. Because of the temporal variation of the magnetic field and the rotation of the cylinder, an electric current is generated in this metallic ring.[5] The volume density $\vec{J}$ of the induced current is given by

$$\vec{J} = \gamma\left[-\frac{\partial\vec{A}}{\partial t} + \vec{V}\times\vec{B}\right]. \tag{2}$$

In Eq. (2), $\vec{A}$ is the magnetic potential vector defined by $\vec{\nabla}\times\vec{A}=\vec{B}$ and $\vec{V}$ is the local velocity. If $\dot{\Theta}$ is the angular velocity of the rotor, then the velocity for a point M of the metallic ring, located at a displacement $\vec{r}$ from the rotation axis, $\vec{r}=x\vec{u}_x+y\vec{u}_y$, is given by $\vec{V}=\dot{\Theta}\ \vec{u}_z\times\vec{r}$, where $\vec{u}_z$ is the unit vector in the vertical direction. It is straightforward to show that $\vec{A}=A\ \vec{u}_z$, where

$$A = yB_x - xB_y = B_0[y\cos(\omega t) - x\sin(\omega t)]. \tag{3}$$

Consequently, if we substitute Eqs. (1) and (3) into Eq. (2), it follows that

$$\vec{J} = \gamma\left[x\frac{\partial B_y}{\partial t} - y\frac{\partial B_x}{\partial t} - \dot{\Theta}(xB_x + yB_y)\right]\vec{u}_z, \tag{4}$$

where B_x and B_y are the components of $\vec{B}$ in the directions $\vec{u}_x$ and $\vec{u}_y$, respectively.

These currents, which are immersed in the magnetic field $\vec{B}$ that generates them, are therefore subjected to a force $\vec{F}$ and a torque $\vec{\Gamma}$ whose volume densities are respectively,

$$\vec{F} = \vec{J}\times\vec{B} = J[B_x\vec{u}_y - B_y\vec{u}_x] \tag{5a}$$

and

$$\vec{\Gamma} = \vec{r}\times\vec{F} = J[xB_x + yB_y]\vec{u}_z. \tag{5b}$$

As a result, the total magnetic torque $\vec{T}_{\mathrm{mag}}$ applied to the metallic ring is given by

$$\vec{T}_{\mathrm{mag}} = T_{\mathrm{mag}}\vec{u}_z = \oint\oint\oint_{\text{metallic ring}} \Gamma(x,y)\,dxdydz. \tag{6a}$$

Based on symmetry considerations, it is straightforward to show that

$$\oint\oint\oint_{\text{metallic ring}} x^2 dxdydz = \oint\oint\oint_{\text{metallic ring}} \times y^2 dxdydz = \pi\frac{\ell_0 a_0^4}{4} = I_0 \quad \oint\oint\oint_{\text{metallic ring}} \times xydxdydz = 0. \tag{6b}$$

As a result, the expression for the total magnetic torque reduces to

$$\vec{T}_{\mathrm{mag}} = \gamma I_0[B_x\dot{B}_y - B_y\dot{B}_x - \dot{\Theta}(B_x^2 + B_y^2)]\vec{u}_z, \tag{7}$$

where I_0 is the intensity of the induced current. If we substitute into Eq. (7) the expression for the rotating magnetic field given by Eq. (1), we obtain an even simpler relation:

$$\vec{T}_{\mathrm{mag}} = \gamma I_0 B^2(\omega - \dot{\Theta})\vec{u}_z. \tag{8}$$

We point out that like asynchroneous motors, the magnetic torque is a maximum when $\dot{\Theta}(t=0)=0$, that is, when the rotor is at rest. As a result, if the frictional torque is not too large, the rotor starts rotating.

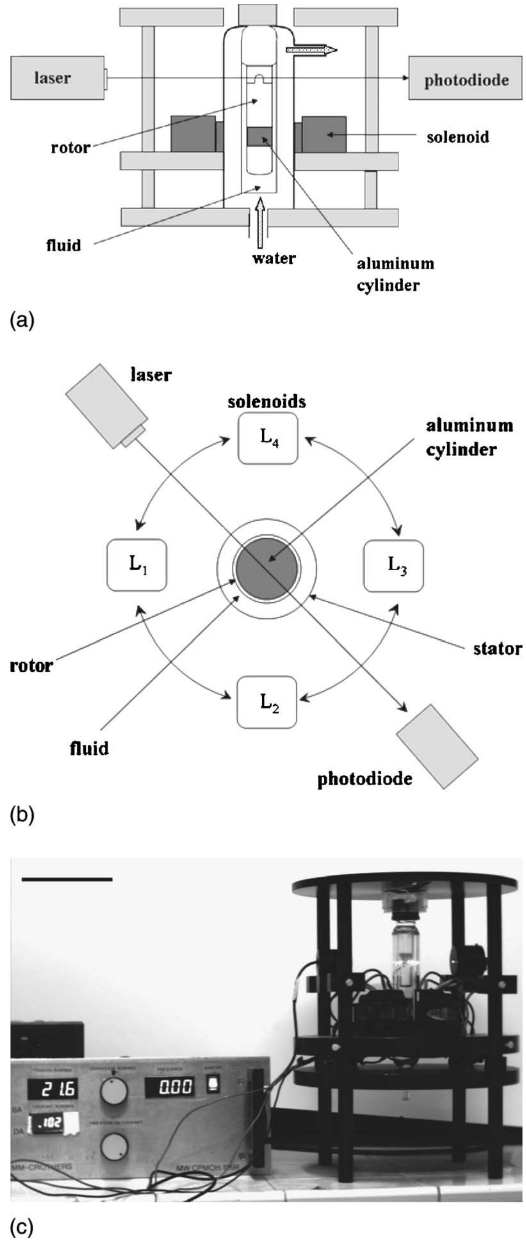


Fig. 1. (a) Schematic side view representation of the viscometer. (b) Schematic top view representation of the viscometer. (c) Photograph image of the viscometer. The length of the black bar is 10 cm.

B. Mechanical action

We now discuss the resulting movement of the rotor when placed in this rotating magnetic field. As mentioned, this rotor consists of an inner glass cylinder and a metallic (aluminum) ring fixed inside (see Fig. 1). Because no mechanical devices are attached to it, it is subject only to the magnetic field and to the forces due to the fluid sandwiched in the annular gap between itself and the fixed outer cylinder, that is, the stator. As shown by Eq. (8), the magnetic torque is positive for $\dot{\Theta}<\omega$, and consequently increases the angular velocity of the rotor; in contrast, the frictional action of the fluid always decreases the angular velocity of the rotor. Consequently, in the steady state, the mechanical action of the magnetic torque, $\vec{T}_{\rm mag}$, is balanced by the torque, $\vec{T}_{\rm fluid}$, resulting from the friction with the fluid sandwiched between both cylinders, namely:

$$\vec{T}_{\rm mag} + \vec{T}_{\rm fluid} = \vec{0}. \tag{9}$$

With this geometry, the fluid flow in the steady state corresponds to Couette flow.[1] The local fluid velocity, which depends only on the distance r to the rotation axis, is given by $\vec{V}(\vec{r}) = r\omega(r)\vec{u}_\theta$, where $\omega(r)$ is the local angular velocity of the fluid, and therefore the local shear rate obeys $\dot{\gamma}(r) = r(d\omega/dr)$. In the steady state, the resulting torque of the fluid friction acting on the volume between the cylinders of radii r and $r+dr$ must be zero, so the local tangential stress obeys the relation: $r^2\sigma(r) = C$, where C is a constant. For a Newtonian fluid, the local tangential stress is $\sigma(r) = \eta\dot{\gamma}(r)$. Therefore, the angular velocity is the solution of the differential equation:

$$\frac{d\omega}{dr} = \frac{C}{\eta r^3}. \tag{10}$$

The solution of Eq. (10) with the two boundary conditions, $\omega(r_1) = \dot{\Theta}$ and $\omega(r_2) = 0$, leads to

$$C = -\eta\dot{\Theta}\frac{2r_1^2 r_2^2}{r_2^2 - r_1^2}, \tag{11a}$$

and

$$\sigma(r) = -\frac{2r_1^2 r_2^2}{r_2^2 - r_1^2}\frac{\eta\dot{\Theta}}{r^2}. \tag{11b}$$

Therefore the frictional torque applied to the surface of the rotor due to the movement of the fluid is given by

$$\vec{T}_{\rm fluid} = -2\pi\,\ell r_1^2\sigma(r_1)\vec{u}_z = \frac{4\pi r_1^2 r_2^2}{r_2^2 - r_1^2}\ell\,\eta\dot{\Theta}\vec{u}_z. \tag{12}$$

We substitute Eqs. (8) and (12) into Eq. (9), and obtain a relation for the fluid viscosity only in terms of the set-up parameters:

$$\eta = \gamma I_0 B^2 \frac{(\omega - \dot{\Theta})}{\dot{\Theta}} \frac{(r_2^2 - r_1^2)}{4\pi r_1^2 r_2^2 \ell}. \tag{13}$$

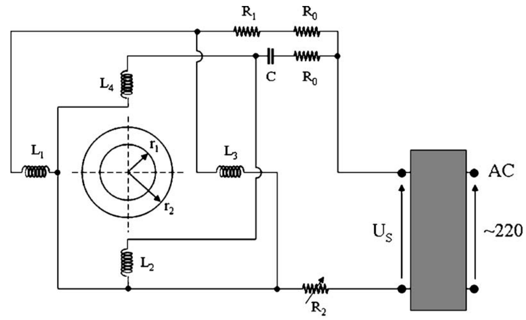
For $\dot{\Theta} \ll \omega$, which is typical for most experiments, the viscosity is well approximated by the following relation:

$$\eta = \gamma I_0 B^2 \frac{\omega}{\dot{\Theta}} \frac{(r_2^2 - r_1^2)}{4\pi r_1^2 r_2^2 \ell}. \tag{14}$$

III. CONSTRUCTION AND CALIBRATION

A. Design

In practice, the rotating horizontal magnetic field is produced by four identical solenoids (AS41152 HS solenoids from Matsushita) placed outside the glass cylinder, at the same distance $d = 25$ mm from its symmetry axis as shown in Figs. 1(b) and 2(b). To obtain a rotating magnetic field, solenoids facing each other are electrically connected as in Fig. 2. The amplitude of the rotating magnetic field is then fixed



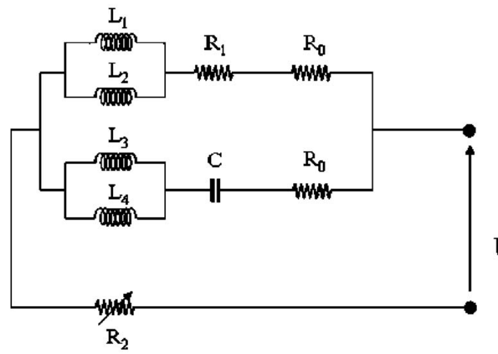


Fig. 2. (a) The electrical circuit driving the viscometer. The value of the intensity is monitored by the value of the voltage delivered by the autotransformer, but also can be varied by changing the resistor value R_2. (b) The equivalent electrical circuit. The values of the capacitor C and of the resistor R_1 are chosen so that the phase shift between the LCR and the LR branches is 90° [see Eq. (15)].

by the value of the intensity of the electric current circulating inside these solenoids. Another version of this instrument relies on rotating a magnetic field formed by permanent magnets glued to a cylinder that is spun at a low frequency by a motor.

In our case, the inductance and the internal resistance values of the solenoids are, respectively, $L = 530$ mH and $r = 124\ \Omega$. This equivalent electrical circuit consists of LCR and LR branches in shunt [see Fig. 2(b)]. To make a circular rotating magnetic field, the electrical currents between both branches must have the same modulus and a 90° phase shift between them. These two conditions are established for

$$R_0 = \frac{L\omega - r}{2}, \quad C = \frac{1}{\omega(R + L\omega)}. \tag{15}$$

When $\omega = 2\pi N = 314$ rad/s (corresponding to the European electrical frequency $f = 50$ Hz), these conditions are fulfilled for $R_0 = 21.25\ \Omega$, $C = 16.6\ \mu$F, and $R = 25.45\ \Omega$. A variable autotransformer with the local supply (220 V in Europe) lets us set and vary the voltage up to 65 V and therefore to control the electrical current up to 300 mA. The inner cylinder is a very light NMR glass tube (514-1PP3 from Wildmad,) whose height, outer diameter, and weight are, respectively, 76 mm, 12 mm, and about 3.2 g. It is placed inside a larger outer cylinder, whose inner diameter is 14 mm. To make the inner cylinder rotate when set in the horizontal magnetic field, we place a 10 mm high aluminum cylinder (weight 4.15 g) inside. For most of the fluids we studied, the inner cylinder is well

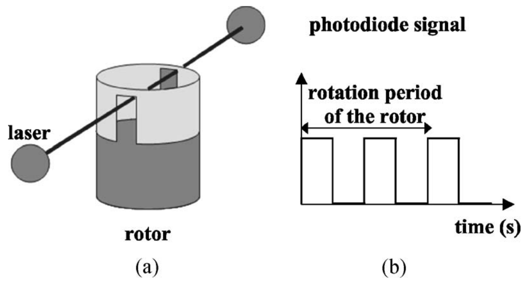


Fig. 3. Schematic representations of (a) the plastic part, which inserted in the rotor cylinder allows us to measure the rotation frequency of the rotor, and (b) the corresponding signal of the photodiode.

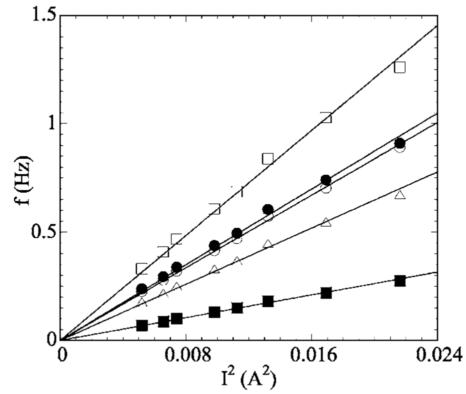
supported by buoyancy. To adjust the meniscus of the interstitial fluid to the top of the rotor precisely, we insert drop by drop some of the same fluid inside the inner cylinder. A water bath regulates and controls the temperature of the outer cylinder and therefore that of the interstitial fluid within 0.1 K accuracy.

The angular velocity of the cylinder rotation, $\dot{\Theta}$, is measured using a low power He–Ne laser and a photodiode. Briefly, the laser beam passes through the axis of the inner cylinder near the top (Fig. 3). When the cylinder rotates, the time variation of the optical signal received by the photodiode placed across the cylinder along the laser path is analyzed. For a constant rotation speed of the cylinder, the signal is periodic, and its frequency corresponds to that of the cylinder rotation. By using a commercial circuit that converts frequency to voltage, the value of the angular rotation can be read directly. This frequency also can be measured by manually timing the angular rotation of the rotor. The results obtained with this latter method also are accurate. It is interesting that, probably due to surface tension, the axis of rotation of the inner (rotating) cylinder is identical to that of the stator.

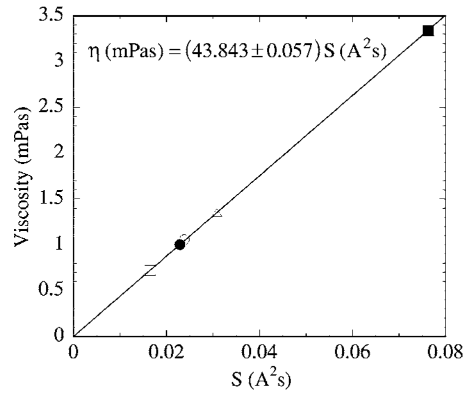
B. Calibration

Figure 4(a) shows the variation of f, the rotational frequency of the rotor, as a function of I^2, where I is the effective intensity of the electrical current circulating in the coils, measured for different Newtonian fluids of known viscosity. As expected from Eq. (14) (because the magnetic field B created by the solenoids is proportional to I), we observe that f varies as I^2. As derived in Eq. (14), S, the slope of I^2 vs f is proportional to η, the fluid viscosity. The coefficient $A = \eta/S$ (where η is given in mPas) is an apparatus constant that can be determined by using different Newtonian fluids of known viscosity [see Table I and Fig. 4(b)].

When the fluid exhibits non-Newtonian behavior, the rotation frequency of the rotor no longer varies as I^2 (see Fig. 5). Because the annular gap cell is small compared to the radius of the glass tubes, the stress in the fluid is nearly constant, and therefore the fluid viscosity can still be well approximated by the relation $\eta = AI^2/f$, which is valid for Newtonian behavior, with $A = (43.843 \pm 0.057)\text{mPa A}^{-2}$ [see Fig. 4(b)]. In this approximation, the value of $\dot{\gamma}$, the shear rate, can be obtained from the measurement of f using the relation $\dot{\gamma} = 2\pi f r_1/e$, where e is the annular gap spacing between the rotor and the stator. For the parameters of our experiment, this relation leads to $\dot{\gamma} = 26\,\pi f \approx 81.7 f$.



(a)



(b)

Fig. 4. (a) The rotation frequencies of the rotor as a function of the square of the electrical current intensity for different Newtonian fluids of known viscosity (see Table I for details). The solid lines correspond to the best linear fit: $f(\text{Hz}) = 1/SI^2(\text{A}^2)$. We find for distilled water, $f = (41.901 \pm 0.309)I^2$ for $T = 18\ °\text{C}$ (open circles), $f = (43.678 \pm 0.523)I^2$ for $T = 20\ °\text{C}$ (closed circles), and $f = (60.575 \pm 0.726)I^2$ for $T = 35\ °\text{C}$ (open squares). For hexadecane at $T = 20\ °\text{C}$ (closed squares) and the dodecane at $T = 25\ °\text{C}$ (open triangles), the best linear fits are, respectively, $f = (13.123 \pm 0.137)I^2$, and $f = (32.403 \pm 0.411)I^2$. (b) The slope $S = I^2/f$ determined from the best linear fit of I^2 vs f as a function of η, the viscosity given by Ref. 7 for the temperature of the experiment. The best linear fit between S and η yields the relation: $\eta(\text{mPas}) = (43.843 \pm 0.057)S(\text{sA}^2)$.

IV. CONCLUSION

Building a Zimm–Crothers viscometer is inexpensive compared to commercial rheometers. Its design is a good illustration of the use of both electromagnetism and fluid mechanics, and it can be done at the undergraduate level. In addition to the performance of such a viscometer, it is an

Table I. The nature of the different fluids and the experimental conditions used for the calibration of the viscometer. The symbols correspond to those used in Fig. 4(a).

Nature of the Fluid	Temperature (°C)	Symbols	Viscosity(mPas) from Handbook
Distilled water	18	Open circles	1.056
Distilled water	20	Closed circles	1.002
Distilled water	35	Open squares	0.7225
Hexadecane	20	Closed squares	3.34
Dodecane	25	Open triangles	1.35

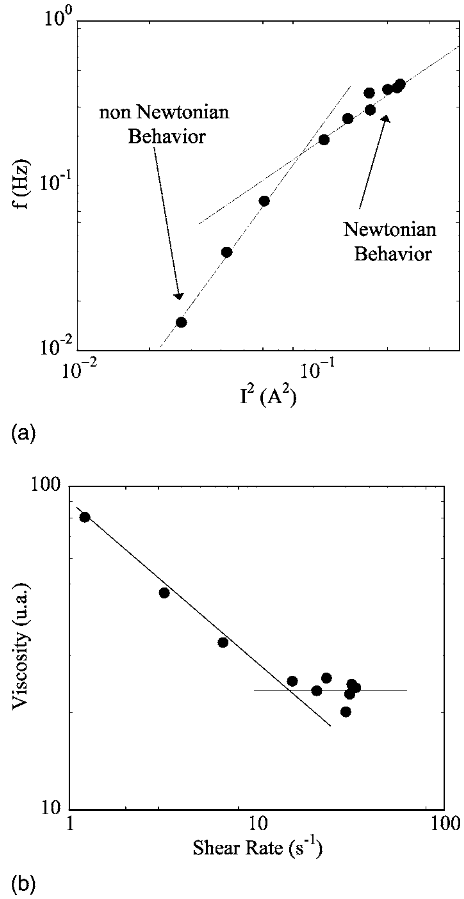


Fig. 5. (a) Plot of I^2 vs f measured a non-Newtonian dilute microemulsion made of 38.65 wt % water (brine NaCl 9 g/l), 2.15 wt % sodium dodecyl sulfate (SDS), 55 wt % decane, and 4.2 wt % octanol at $T=23.8$ °C. (Ref. 6). (b) Dependence of the corresponding viscosity vs the shear rate.

ideal tool for accurately measuring the shear viscosity (and its temperature dependence) for low viscosity Newtonian and non-Newtonian liquids. Illustrative examples have been given for both cases, and the precision on the viscosity measurements is better than 5%.

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