Problem 1. (Cosets [AM106]) Let \( H = \{e, (12)(34), (13)(24), (14)(23)\} \leq S_4. \)

1. List the left-cosets of \( H \) in \( S_4 \).

2. We can also view \( H \) as a subgroup of \( S_6 \). How many left-cosets does \( H \) have in \( S_6 \)?

Problem 2. (Subgroups of \( \mathbb{C}^* \)) Determine all of the finite subgroups of \( \mathbb{C}^* \). Justify your answer. (Hint: what are the solutions to \( a^n = 1 \) in \( \mathbb{C}^* \)?)

Problem 3. (Orbits and Stabilizers for the Cube) Let \( G \) be the group of rotational symmetries of a regular cube in \( \mathbb{R}^3 \). (We do not include reflections in \( G \).)

1. Among points \( s \) on the surface of the cube (including edges and corners), what are the possible orbit sizes? For each answer \( a \) you give, provide an example of a point \( s \) with with \( |\text{orb}_G(s)| = a \).

2. For each point \( s \) above, describe \( \text{stab}_G(s) \).
Problem 4. (Classification of Abelian Groups [AM106-A]) Determine which of the following groups are isomorphic to each other:

1. $\mathbb{Z}_{40}$.
2. $\mathbb{Z}_8 \times \mathbb{Z}_5$.
3. $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5$.
4. $\mathbb{Z}_{55}^*$.
5. $\mathbb{Z}_{88}^*$.
6. $\mathbb{Z}_{100}^*$.

Problem 5. (Richness of $\mathbb{Z}_n^*$ [AM206-A]) Dirichlet’s Theorem says that if $a$ and $b$ are relatively prime integers, then the arithmetic progress $\{a + tb : t \in \mathbb{Z}\}$ contains infinitely many prime numbers. Use this to show that for every finite abelian group $G$, there is an $n \in \mathbb{N}$ such that $G \leq \mathbb{Z}_n^*$. (This result is similar in spirit to Cayley’s Theorem, which says that for every finite group (even non-abelian), there is an $n \in \mathbb{N}$ such that $G \leq S_n$.)