

Problem Set 0

Assigned: Wed. Aug. 30, 2017

Due: Fri. Sept. 1, 2017 (11:59 pm)

- This problem set is optional and will not count for your grade. However, if you have not taken a prior proof-based math course, it is strongly encouraged that you complete and turn in the problem set for practice and feedback on doing proofs.
- You should submit your solutions via assignment page on the canvas website of the course.

Problem 1. (Proof by Contradiction) Joe the painter has 2016 cans of paint. Show that at least one of the following statements is true about Joe's paint collection.

- Among the cans, there are at least 32 of them with the same color.
- Among the cans, there are at least 66 different colors of paint.

Problem 2. (Set Equality) Which of the following is true? Prove your answers.

- For every three sets A, B, C , we have $A \cup (B \cap C) = (A \cup B) \cap C$.
- For every three sets A, B, C , we have $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Problem 3. (Induction) The Fibonacci numbers F_0, F_1, \dots are defined inductively by $F_0 = 0$, $F_1 = 1$, and $F_{n+2} = F_{n+1} + F_n$ for all $n \geq 1$. Thus the sequence (starting at F_0) is $0, 1, 1, 2, 3, 5, 8, 13, 21, \dots$. Prove by induction that for $n \geq 2$, $F_n \geq \varphi^{n-2}$, where $\varphi = (1 + \sqrt{5})/2$ is the golden ratio.

Problem 4. (Incorrect Induction) What is the wrong with the following proof by induction?

Claim: In every set of n students, all students have the same height.

"Proof" by Induction:

- Base Case: For every set of size 1, the claim is clearly true (all the students in that set have the same height).
- Induction Step: Assume that the claim is true for sets of k students (this is the induction hypothesis), and we'll prove that it also holds for sets of $k + 1$ students. Consider an arbitrary set S consisting of $k + 1$ students, say $S = \{p_1, \dots, p_{k+1}\}$. Let $S' = \{p_1, \dots, p_k\}$. Since $|S'| = k$, our induction hypothesis tells us that all students in S' have the same height. So now we only need to show that p_{k+1} has the same height too. To do this, consider the set $S'' = \{p_2, \dots, p_{k+1}\}$. Since $|S''| = k$, the induction hypothesis also tells us that all students in S'' have the same height. In particular, p_{k+1} has the same height as p_2 , and hence the same height as all students in S' .