

## Problem Set 5

Assigned: Wed. Oct. 11, 2017

Due: Wed. Oct. 18, 2017 (8:00 AM)

- You may submit your solutions via assignment page on the canvas website of the course.
- For collaboration and late days policy, see course website at <http://madhu.seas.harvard.edu/courses/Fall12017>
- Aim for clarity and conciseness in your solutions, emphasizing the main ideas over low-level details. Justify your answers except when otherwise specified.

**Problem 1. (Cosets)** Let  $H = \{e, (12)(34), (13)(24), (14)(23)\} \leq S_4$ .

1. List the left-cosets of  $H$  in  $S_4$ .
2. We can also view  $H$  as a subgroup of  $S_6$ . How many left-cosets does  $H$  have in  $S_6$ ?

**Problem 2. (Subgroups of  $\mathbb{C}^*$ )** Determine all of the finite subgroups of  $\mathbb{C}^*$ . Justify your answer. (Hint: what are the solutions to  $a^n = 1$  in  $\mathbb{C}^*$ ?)

**Problem 3. (Orbits and Stabilizers for the Cube)** (See notes for Lecture 8 for definitions of Orbits and Stabilizers.) Let  $G$  be the group of rotational symmetries of a regular cube in  $\mathbb{R}^3$ . (We do not include reflections in  $G$ .)

1. Among points  $s$  on the surface of the cube (including edges and corners), what are the possible orbit sizes? For each answer  $a$  you give, provide an example of a point  $s$  with  $|\text{orb}_G(s)| = a$ .
2. For each point  $s$  above, describe  $\text{stab}_G(s)$ .

**Problem 4. (Classification of Abelian Groups)** Determine which of the following groups are isomorphic to each other:

1.  $\mathbb{Z}_{40}$ .
2.  $\mathbb{Z}_8 \times \mathbb{Z}_5$ .
3.  $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_5$ .
4.  $\mathbb{Z}_{55}^*$ .
5.  $\mathbb{Z}_{88}^*$ .
6.  $\mathbb{Z}_{100}^*$ .