Solution 1:

This NFA recognizes the desired language.

Solution 2:

We will prove this by contradiction using the Pumping Lemma.

Let $P$ denote the language of strings which are palindromes. Suppose $P$ is regular, and $M$ is a DFA which recognizes $P$.

Consider the input $0^p110^p$ where $p$ is the pumping length. Write $p = xyz$ according to the pumping lemma. We know from the pumping lemma that $|xy| \leq p$. The first $p$ symbols of the input string are all 0’s, so we know that $y$ is a string of $k$ zeros for some $k > 0$.

By the pumping lemma, the string $xyyz$ must be contained in the language $P$. However, $xyyz = 0^{p+k}110^p$, which is not a palindrome. This is a contradiction, so $P$ cannot be regular.

Solution 3:

Suppose DFA $M$ over states $Q$ recognizes the language $A$. We will construct an NFA $M'$ over states $Q'$ which recognizes $A'$.

Let $Q' = Q \cup \{q_{start}\}$

$q_{start}$ will serve as the initial state for $M'$, while $q_0 \in Q$ will denote the initial state of $M$

Let $\delta(q_0, 0)$ and $\delta(q_0, 1)$ represent the states occupied by $M$ after reading 0 or 1 as the first input symbol, respectively.

The transition function of $M'$ will be the same as that of $M$, except $\delta'(q_{start}, 0) = \{\delta(q_0, 1), \delta(q_0, 1)\}$. Graphically, this means that $q_{start}$ has edges labelled 0 pointing to both $\delta(q_0, 0)$ and $\delta(q_0, 1)$. $q_{start}$ has no other outward edges.

If the empty string is in $A$, let $q_{start}$ be an accepting state. Otherwise $q_{start}$ is not an accepting state.
Consider a non-empty string $w$ such that $w \in A$ and let $f(w)$ be the same string where the first character is replaced with 0.

If $w_1 = 0$, that means starting DFA $M$ at state $\delta(q_0, 0)$ with the input $w_2, w_3, ..., w_k$ results in an acceptance. Consider the action of DFA $M'$ on $f(w)$. After consuming the initial 0 at the start of the string, one copy of $M'$ occupies state $\delta(q_0, 0)$ with the symbols $w_2 w_3, ..., w_k$ left to process. This copy of the NFA will end up in an accepting state, and $M'$ will accept $f(w)$.

Analogously, if $w_1 = 1$, one copy of $M'$ will occupy state $\delta(q_0, 1)$ with the symbols $w_2 w_3, ..., w_k$ left to process, resulting in an acceptance. Therefore any string in $A'$ is recognized by $M'$.

To show that no other strings are recognized by $M'$, consider a non-empty input $0w_2 ... w_k$ which is accepted by $M'$. For this input to be accepted by $M'$, the DFA $M$ must accept after starting in $\delta(q_0, 0)$ with input $w_2 ... w_k$, or it must accept after starting in $\delta(q_0, 1)$ with input $w_2 ... w_k$. This means that the DFA $M$ would accept at least one of $1w_2, ..., w_k$ or $0w_2 ... w_k$, so $0w_2 ... w_k \in A'$.

Note that, because we have chosen whether to make $q_{\text{start}}$ an accepting state based on whether the empty string is in $A$, we know $\epsilon$ is accepted by $M'$ if and only if $\epsilon \in A$, which also means $\epsilon \in A'$.

Illustration: