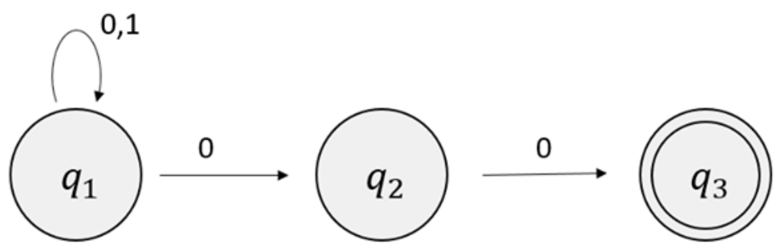
CS 121 Section 4 Solutions: Fall 2020

Solution 1:



This NFA recognizes the desired language.

Solution 2:

We will prove this by contradiction using the Pumping Lemma.

Let P denote the language of strings which are palindromes. Suppose P is regular, and M is a DFA which recognizes P.

Consider the input 0^p110^p where p is the pumping length. Write $p = xyz$ according to the pumping lemma. We know from the pumping lemma that $|xy| \leq p$. The first p symbols of the input string are all 0's, so we know that y is a string of k zeros for some $k > 0$.

By the pumping lemma, the string $xyyz$ must be contained in the language P. However, $xyyz = 0^{p+k}110^p$, which is not a palindrome. This is a contradiction, so P cannot be regular.

Solution 3:

Suppose DFA M over states Q recognizes the language A. We will construct an NFA M' over states Q' which recognizes A'.

Let $Q' = Q \cup \{q_{start}\}$
q_{start} will serve as the initial state for M', while $q_0 \in Q$ will denote the initial state of M
Let $\delta(q_0, 0)$ and $\delta(q_0, 1)$ represent the states occupied by M after reading 0 or 1 as the first input symbol, respectively.
The transition function of M' will be the same as that of M, except $\delta'(q_{start}, 0) = \{\delta(q_0, 1), \delta(q_0, 1)\}$.
Graphically, this means that q_{start} has edges labelled 0 pointing to both $\delta(q_0, 0)$ and $\delta(q_0, 1)$. q_{start} has no other outward edges.
If the empty string is in A, let q_{start} be an accepting state. Otherwise q_{start} is not an accepting state.

Consider a non-empty string w such that $w \in A$ and let $f(w)$ be the same string where the first character is replaced with 0.
If $w_1 = 0$, that means starting DFA M at state $\delta(q_0, 0)$ with the input $w_2, w_3, ..., w_k$ results in an acceptance. Consider the action of DFA M' on $f(w)$. After consuming the initial 0 at the start of the string, one copy of M' occupies state $\delta(q_0, 0)$ with the symbols $w_2 w_3, ... w_k$ left to process. This copy of the NFA will end up in an accepting state, and M' will accept $f(w)$.
Analogously, if $w_1 = 1$, one copy of M' will occupy state $\delta(q_0, 1)$ with the symbols $w_2 w_3 ... w_k$ left to process, resulting in an acceptance. Therefore any string in A' is recognized by M'.

To show that no other strings are recognized by M', consider a non-empty input $0w_2...w_k$ which is accepted by M'. For this input to be accepted by M', the DFA M must accept after starting in $\delta(q_0, 0)$ with input $w_2...w_k$, or it must accept after starting in $\delta(q_0, 1)$ with input $w_2...w_k$. This means that the DFA M would accept at least one of $1w_2, ...w_k$ or $0w_2...w_k$, so $0w_2...w_k \in A'$.

Note that, because we have chosen whether to make q_{start} an accepting state based on whether the empty string is in A, we know ϵ is accepted by M' if and only if $\epsilon \in A$, which also means $\epsilon \in A'$.

Illustration: