CS 221 Computational Complexity, PSet 2 **Due: Feb 16, 2018 at 8pm**

Problem Set 2

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1. **Robustness of NC1.**
   For a function $f : \{0,1\}^n \to \{0,1\}$, prove the following are equivalent:
   (a) f has a logarithmic depth circuit.
   (b) f has a log depth formula.
   (c) f has a polynomial sized formula.
   (d) f has a log depth arithmetic forumla: i.e., a formula with binary XOR ($\mathbb{F}_2$-sum), and AND ($\mathbb{F}_2$-multiplication) gates with some inputs being allowed to be the constant 1.

2. **Path vs co-Path.**
   Give a logspace reduction for the following transform:
   - **Input:** (G, s, t); a directed graph G and two vertices s, t.
   - **Output:** (G', s', t') such that there is an $s' \to t'$ path in G' if and only if there is **no** $s \to t$ path in G.

   If G has n vetices, how many vertices does G' have?

3. **Universal Algorithm.**
   Give an explicit algorithm for Factoring such that the running time of this algorithm is polynomial if and only if Factoring can be solved in polynomial time.

4. **Hierarchy for Circuit Size.**
   Show that $SIZE(n) \neq SIZE(n^2)$. (What is the tightest such hierarchy you can show?)

5. **Circuit Lower Bounds.**
   Show that EXPSPACE $\not\subseteq$ SIZE$(2^n/(100n))$.

6. **Formulas vs. Bounded width branching programs. (Optional/Extra Credit).**
   This problem is "extra credit." It does not count towards your grade on the problem set. Solve it only to express your interest in the material. No collaboration/discussion allowed, and no looking up external websites/references. Writeup is as important as the idea behind your solution.

   Show that a function $f : \{0,1\}^n \to \{0,1\}$ has a polynomial sized formula if and only if it has a width-8 branching programs of polynomial size.

   For the harder direction, you may try to show the following: Given an arithmetic formula f of depth d, produce a "3-register straight-line program" of length 4^d that maps binary input registers (A, B, C) to $(A, B, C + f(x_1....x_n) \cdot B)$, where each line of the straightline program is of the form $(A, B, C) \mapsto (A', B', C')$ with two of $\{A', B, C'\} = \{A, B, C\}$ (say $A' = A$ and $C' = C$) and the third one is a linear form, e.g. $B' = B + x_i \cdot A$.