Today

**AMORTIZED COMMUNICATION**

* Interactive Correlated Sampling

Warning: Will refer to notes from lecture 16 for some parts

Definitions from last time

1. \( f^{\otimes n} : \mathcal{X}^n \times \mathcal{Y}^n \rightarrow \mathbb{R}^n \)
   \[ f(x_1^n, y_1^n) = (f(x_1, y_1), \ldots, f(x_n, y_n)) \]

2. \( \Pi \) computes \( f^{\otimes n} \) with error \( \epsilon \) if
   \[ \forall i \quad \Pi(x_1^n, y_1^n)_i = f(x_i, y_i) \quad \text{w.p.} \geq 1 - \epsilon \]

3. \( CC^\epsilon_n(f) = \min_{\Pi: \Pi \text{ solves } \Pi^{\otimes n}} \mathbb{E}_{(x_1^n, y_1^n)}[\epsilon^2] \)
   \( CC_n^\epsilon(f) = \min_{\Pi: \Pi \text{ solves } \Pi^{\otimes n}} \mathbb{E}_{(x_1^n, y_1^n)}[\epsilon^2] \)
   \( IC_n^\epsilon(f) = \min_{\Pi: \Pi \text{ solves } \Pi^{\otimes n}} \mathbb{E}_{(x_1^n, y_1^n)}[\epsilon^2] \)
   \( IC_n^\epsilon(f) = \min_{\Pi: \Pi \text{ solves } \Pi^{\otimes n}} \mathbb{E}_{(x_1^n, y_1^n)}[\epsilon^2] \)
Theorem [Braverman-Rao]

\[ \forall \mu \quad CC_{\epsilon,\mu}^n(t) = IC_{\epsilon,\mu}^n(t) \cdot (1 + o_n(1)) \]

1. \[ IC_{\epsilon,\mu}^n(f) = n \cdot IC_{\epsilon,\mu}^1(f) \]

Proof: Suppose \( \Pi \) solve \( f \) with error \( \epsilon \), then \( \Pi \circ \text{on} \) solves \( f \text{ on } \text{on} \) with error \( \epsilon \).

\[ IC(\Pi \circ \text{on}) = n \cdot IC(\Pi) \]

2. \[ IC_{\epsilon,\mu}^n(f) \geq n \cdot IC_{\epsilon,\mu}^1(f) \]

More Crucial Step:

Corollary:

\[ CC_{\epsilon,\mu}^n(t) \geq IC_{\epsilon,\mu}^n(f) \geq n \cdot IC_{\epsilon,\mu}^1(f) \]

Need to show:

\[ IC_{\epsilon,\mu}^1(f) \leq n \cdot IC_{\epsilon,\mu}^1(f) \cdot (1 + o(1)) \]

[New "compression protocol"]
Interactive Correlated Sampling

Alice $\leftrightarrow P \xrightarrow{\epsilon} Q \rightarrow Bob$

$D(P || Q) + O(D(P || Q) + \log \frac{1}{\epsilon})$

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**Key Idea**

- Suppose $C = \frac{Q(x_i)}{P(x_i)}$ known [very unreasonable!]

- Alice outputs first $x_i$ s.t. $(x_i, a_i)$ satisfies $a_i < P(x_i)$

- Bob scale $Q$ by factor $C$

- Now needs to output one of the set

  $$S = \{ (x_j, a_j) | a_j < C \cdot Q(x_j) \}$$

- Which one?

- Alice sends $(\log C + O(1))$ bits of hash of $x_i$ to Bob

- Hopefully isolates identities $x_i$ uniquely in $S$.

  Alice outputs the unique $x_i$ element.
Final Compression Protocol

- Compress each round to $D(P_v^A || P_v^B) / n$ on $i$th level

- Total comm. = $D(\mathbf{P} || \mathbf{Q}) + O(\sqrt{\mathbf{IC} + \log 1/\varepsilon})$

  = $\mathbf{IC} + O(\sqrt{\mathbf{IC} + \log 1/\varepsilon})$. \[\]