Today: Distance Amplifying Codes

aka “ABNNR Construction”

“AEL Construction”

“GI results”

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Error in PS 4.4
Part 6 omitted

- Weekly Reports.
- OH today + Friday (Chining)
- PS 4 next Friday
- PS 3 Due yesterday
The ABNNR Construction
- takes a weak ECC (low distance)
  \[ \Rightarrow \text{converts it to a strong one} \quad \text{(very high distance)} \]
- Start with a bipartite expander \( B = (L, R, E) \)
  \[ |L| = |R| \]
  \( d \)-regular

\[ X = (x_1, \ldots, x_n) \in \mathbb{F}_2^n \quad \downarrow \]

\[ Y = (y_1, \ldots, y_n) = \left( \mathbb{F}_2^d \right)^n \]

\[ x \mapsto y \quad \text{not a good error correc. code} \]
Fix: only use $x = (x_1 \ldots x_n) \in C_0 \subseteq \mathbb{F}_2^n$ with $C_0$ having modest distance.

- Can fix $C_0$ s.t. $S(C_0) = 0.1$
  
  $R(C_0) = \frac{1}{2}$

- Will use $d$ very large (much larger than $\frac{1}{S(C_0)}$)

  \[ \Rightarrow \text{ then encoding } f \mapsto (f(x_1) \ldots f(x_n)) \]

\[ \oplus \]

\[ \text{Then } \exists \text{ graph } B \text{ s.t. } \forall \text{ subset of left right of size } \geq 0.1 n \]

\[ \Rightarrow \text{ see at least } (1 - \frac{1}{d})^n \text{ neighbors on right.} \]

\[ \Rightarrow \# y \text{ is a code whose distance is } \approx 1 - \frac{1}{d} \]

\[ \Rightarrow \text{ can push better } C_0 \]

\[ \Rightarrow \text{ rate } \frac{1}{d} \]

\[ \text{Rate}(C) = \frac{1}{2} \]

\[ \text{Rate}(C) = \frac{1}{2d} \]

\[ S(C) = 1 - \frac{1}{d} \]
Application: Use these codes $+$ concatenation to get binary codes of distance $\frac{1}{2} - \varepsilon$

$k$ message bits to $O\left(\frac{k}{\varepsilon^3}\right)$ codeword bits

Yet...

Codes are of small rate

Rate $= \Omega(\varepsilon^3)$

Here is AEL comes in
- What is the rate of this process?
  - Preceding loss $\rightarrow 1 - o(1)$
  - $l$ bits $\rightarrow d$ bits $\rightarrow R = \frac{l}{d}
\sum \frac{1}{d} - o(1)$
  - Moved bits around $\rightarrow$ rate $= 1$
- What is the distance? Alphabet $= \mathbb{F}_{2d}$

Typical vertex on right should see
$
\approx \frac{|S|}{n} \cdot d
$
neighbors in $S$.

\[
|\bigcap_{u \in U} U_T| \leq (1 - (8 - 1)) \cdot d
\leq |S| \leq \frac{1}{8} 
\]
Want from graph $B$:

size of largest $T$ s.t.

$$
\prod \left( T \right) \leq \sum_{u \in L} \left| \prod(u) \cap \prod(T) \right| \leq (1-\delta)d
$$

then

$$
\left| \prod \left( T \right) \right| \geq (0.01)n
$$

For graphs $s.t. \text{ size of } T \text{ is small}

$\Rightarrow \ldots \text{ codes of } R-o(1), \text{ distance } S\ldots$
Guruswami - Indyk: Linear-time decoding based on this construction.

- Pick $B$ - random graph -- .

$\left\{ \begin{align*}
\text{will my construction work?} \\
\text{algorithm work?}
\end{align*} \right.$

- Extract right graph-theoretic properties that made the proof work & use explicit graphs.
Will attempt in PS5: to use GI idea
to repair PS4.4⑤
⇒ alphabet-reduction in FRS
& get list-decoding close to capacity
over $O(1)$ alphabets.

Let's sized

on nd edges