ABSTRACT

Coherent change detection using paired synthetic aperture radar (SAR) images is typically performed using a classical estimator of coherence applied under an assumption of complex Gaussian data. The magnitudes of the resultant coherence estimates are plotted as an image and used to gauge changes in the observed scene. Here we investigate the suitability of an alternative coherence estimator that further assumes the variances of the populations underlying each paired sample to be equal. We show experimentally that this alternative estimator outperforms the classical estimator even when the underlying variances are not equal, as long as they are close enough. We demonstrate the suitability of this estimator directly on publicly available synthetic aperture radar data, with a performance improvement observed through increased contrast in the corresponding coherent change detection images.

Index Terms—Coherence estimation, coherent change detection, interferometric SAR processing, synthetic aperture radar.

1. INTRODUCTION

Synthetic aperture radar (SAR) is an important modality in remote sensing due to its ability to form high resolution images with relative invariance to weather and lighting conditions. SAR images are formed using a moving radar that collects data over a scene from multiple perspectives. The resultant data are complex, with the magnitude corresponding to the reflected signal intensity of the scene and the phase indicating scattering properties.

One application of SAR is coherent change detection (CCD), which utilizes two SAR data collections of the same scene at different times to infer changes that have occurred in between data collections. Local changes in SAR image magnitude can indicate large-scale changes, such as the appearance of a sizeable object during the second collection that was not present during the first. However, since coherent SAR data contain phase as well as magnitude information, a particularly sensitive form of change detection can also be performed. If the two image collections use identical collection geometries such that the respective image phases are aligned, then this information leads naturally to the detection of smaller-scale changes.

As [1, 2, 3] have investigated, the traditional coherence magnitude estimator for SAR imagery is biased, particularly when the true coherence is small. This bias can be reduced by an increase in the number of samples. However, in practice, there are a limited number of samples to be obtained from each spatial location in a pair of SAR images, as they must be “borrowed” from a local neighborhood or spatial window. As the number of neighboring pixels used to estimate coherence is increased, the effective spatial resolution of the resultant CCD image is decreased, making detection of small-scale changes more difficult. Furthermore, as the size of the sample window increases, assumptions that the samples are drawn independently from the same distribution are less likely to be met. Accurate estimation of coherence from a limited number of samples is a challenging problem, which must be overcome either through better models for the data or more accurate estimators.

Here we adopt the latter approach and investigate the classical coherence estimator in comparison to an alternative estimator analyzed by Berger [2] under the assumption of equal population variances. We use simulated data with known truth to compare probability of detection and probability of false alarm using each of two estimators to detect change. Furthermore, we study the effect of unequal variance on the Berger estimator, and we show that when the two variances are unequal but close, the Berger estimator outperforms the classical one. Finally, we apply the two estimators to a publicly available SAR image data example to show the increase in coherence contrast (indicating less bias) using the Berger estimator.

2. STATISTICAL CHANGE DETECTION

SAR images are assumed to be collections of spatially uncorrelated pixels drawn from a zero-mean circularly complex Gaussian distribution. Even homogeneous regions in SAR images exhibit natural variability, as shown by the example pair of SAR magnitude images illustrated in Fig. 1. The statistic used to estimate change in an image pair such as this, corresponding to the estimated coherence between a pair of SAR observations, is thus a random variable depending on the true underlying coherence as well as the number of samples employed in coherence estimation.

Given two spatially registered and zero-mean circularly complex Gaussian SAR images such as those shown in Fig. 1 (only magnitude is displayed), the covariance matrix corresponding to a given pixel location, \( X = [f^H, g^H] \in \mathbb{C}^2 \), is defined by:

\[
\Sigma = \mathbb{E}(XX^H) = \begin{bmatrix} \sigma_f^2 & \rho \sigma_f \sigma_g \\ \rho \sigma_f \sigma_g & \sigma_g^2 \end{bmatrix},
\]

where

\[
\rho = \frac{\mathbb{E}(fg^H)}{\sqrt{\mathbb{E}(f^2)\mathbb{E}(g^2)}} = \frac{\mathbb{E}(fg^H)}{\sigma_f \sigma_g}
\]

is the complex correlation coefficient of the image pair and \( \bar{\rho} \) denotes its complex conjugate. Estimation of this covariance matrix can be
written in terms of $N$ independent replicates (in practice obtained from spatial neighborhoods of $f$ and $g$ in the respective SAR images) as

$$\hat{\Sigma} = \frac{1}{N} \sum_{k=1}^{N} f_k g_k^H. \quad (3)$$

In the classical approach to coherence estimation, the magnitude of the sample complex correlation coefficient is frequently used as the coherence estimator and to detect change regions between two complex SAR images. The classical estimator (here denoted $\hat{\rho}_c$) of the complex correlation coefficient of the image pair can be defined in terms of the random variables $\hat{\Sigma}_{11}, \hat{\Sigma}_{22}$, and $\hat{\Sigma}_{12}$ of the Hermitian positive semi-definite matrix, $\hat{\Sigma}$:

$$\hat{\rho}_c = \frac{\hat{\Sigma}_{12}}{\sqrt{\hat{\Sigma}_{11} \hat{\Sigma}_{22}}}. \quad (4)$$

The corresponding probability density function of $|\hat{\rho}_c|$ for a general repeat-pass image pair with true underlying coherence $\rho$ takes the form

$$p(|\hat{\rho}_c|; |\rho|, N) = 2(N - 1)(1 - |\rho|^2)^{N-2} \cdot F_1(N; N; 1; |\rho|^2, |\hat{\rho}_c|^2), \quad (5)$$

where $F_1(\cdot; \cdot; \cdot; \cdot)$ is the Gauss hypergeometric function. Examples of the distribution of $|\hat{\rho}_c|$ for given $N$ and $\rho$ are shown in Fig. 2. Notice that the bias increases with decreasing $\rho$; this is a well-known problem in the SAR literature, as low coherence corresponds to potential change, which is of particular interest.

Note that as a single pair of SAR images is assumed available, only a limited number of samples is available to estimate coherence in practice. Increasing the size of the spatial neighborhood used to estimate coherence can have unintended effects, serving to smooth the eventual CCD image and resulting in missed change detection. By assuming the underlying population variances remain unchanged (up to scale) between a registered pair of SAR images, however, one can naturally improve the stability of coherence estimation and reduce its bias.

### 2.1. Estimating Coherence Assuming Equal Variance

Coherent change detection techniques aim primarily to detect small, subtle changes between two SAR images. As the two SAR images are of the same scene, it is reasonable to assume the underlying variances of the two sets of image pixels are the same (up to scale). If the underlying variances have significantly changed, that would indicate a large-scale change has taken place that would alter the appearance of the magnitude SAR image, and indeed this is a typical means of incoherent change detection. As shown in Fig. 1, the majority of the SAR magnitude values between the image pair is similar, indicating equal underlying variances. In this subsection, we take advantage of this equal variance assumption to calculate an improved coherence estimate with less bias especially in dealing with a small spatial neighborhood.

Assuming equal variance in a SAR image pair, the natural estimator of the complex correlation coefficient $\rho$, investigated by Berger [2] and denoted $\hat{\rho}_b$, can be written as a function of the elements of $\hat{\Sigma}$ as

$$\hat{\rho}_b = \frac{2 \hat{\Sigma}_{12}}{\hat{\Sigma}_{11} + \hat{\Sigma}_{22}}. \quad (6)$$

Notice that the denominator contains a sum rather than a product of two random variables, suggesting a more stable estimator. Furthermore, since both variances are assumed to be equal, the number of samples used to estimate the true variance is effectively doubled. In scenes where most of the underlying variances remain unchanged, this estimator can be expected to offer improved properties over the classical estimator $\hat{\rho}_c$ of (4).
The expression for the joint probability density function of estimated coherence magnitude $|\hat{\rho}|$ and phase $\theta_{\hat{\rho}}$, for $0 \leq |\hat{\rho}| \leq 1$, $-\pi \leq \theta_{\hat{\rho}} \leq \pi$, is derived by several authors, including in [2]:

$$p(|\hat{\rho}|, \theta_{\hat{\rho}}; \rho, N) = C \frac{|\hat{\rho}|(1 - |\hat{\rho}|)^{2N-3}}{[1 - |\hat{\rho}| |\rho| \cos(\theta_{\hat{\rho}} - \theta_{\rho})]^{2N}}$$  \hspace{1cm} (7)

where

$$C = (1 - |\rho|^2)^N \Gamma(2N)[\pi^{\frac{1}{2}} \Gamma(N) \Gamma(N - \frac{1}{2})]^{2N-1}.$$  \hspace{1cm} (8)

Here, $\hat{\rho} = |\hat{\rho}| e^{i\theta_{\hat{\rho}}}$, and $\rho = |\rho| e^{i\theta_{\rho}}$ is the true underlying complex correlation coefficient as per (2), assuming equal variances such that $\sigma^2_1 = \sigma^2_2$ in (1).

For the problem of change detection, we are most interested in investigating the special case of $\rho = 0$, which implies complete decoherence and corresponding change in an imaged scene. When the true coherence is zero, (7) simplifies to

$$p(|\hat{\rho}|; \rho = 0, N) = \frac{\Gamma(2N) |\hat{\rho}|(1 - |\hat{\rho}|)^{2N-3}}{\pi^{\frac{1}{2}} \Gamma(N) \Gamma(N - \frac{1}{2})^{2N-1}}.$$  \hspace{1cm} (9)

This distribution may be simplified further, in that $|\hat{\rho}|^2$ is equal in distribution to a Beta$(1, N - 1/2)$ random variable. Similarly, $|\hat{\rho}_c|^2$ is Beta$(1, N - 1)$ distributed when $\rho = 0$, from which we conclude that the former is more concentrated near the true coherence value of zero. Figure 3 illustrates this fact by showing a comparison of these two densities for the case $N = 4$. Thus, in the case where the true underlying variances $\sigma^2_1$ and $\sigma^2_2$ are equal and the coherence is zero, we gain from using $|\hat{\rho}|^2$ rather than $|\hat{\rho}_c|^2$. A natural question is to what extent this remains true even as the true underlying variances fail to satisfy this hypothesis—a question we address experimentally below.

### 3. EXPERIMENTAL RESULTS

We first performed a simulated experiment with known ground truth to compare the performance of the two estimators $|\hat{\rho}|$ and $|\hat{\rho}_c|$ in detecting change. For the purposes of this experiment, $\rho = 0$ was chosen to indicate change, and $\rho = 0.9$ to indicate no effective change. Note that choosing $\rho = 1$ would result in no variability between samples, which is not realistic in SAR data. Coherence is affected by factors other than scene change, making $\rho = 0.9$ a reasonably high coherence value. We investigated the robustness of $|\hat{\rho}|$ to a violation in the assumption of equal variances, by keeping their sum equal but altering their ratio $R$ over a range of values. Results in detecting change corresponding to $\rho = 0$ versus $\rho = 0.9$ were obtained using $10^5$ independent Monte Carlo trials, for sample sizes $N = 3$ and $N = 6$. These values of $N$ were chosen to reflect realistic spatial window sizes in SAR CCD, where a neighborhood of nine spatially dependent pixels (from a $3 \times 3$ window) is typically modeled as a set of independent observations, but with $N < 9$ to reflect a smaller effective sample size.

Figures 4(a) and (b) show the receiver operating characteristic (ROC) curves for coherence estimators with varying variance ratios $R$, with sample sizes of $N = 3$ and $N = 6$ respectively. The classical coherence magnitude estimator $|\hat{\rho}|$ is denoted as a blue line, and $|\hat{\rho}_c|$ is represented with a red line. As shown in Fig. 4(a), our empirical studies indicate that for $N = 3$, when the variance ratio is...
greater than $R = 0.6$, $|\hat{\rho}|$ outperforms $|\hat{\rho}_b|$. The changeover point of $R = 0.6$ indicates that, at least under these experimental conditions, $|\hat{\rho}_b|$ is fairly robust to deviations from the assumption of equal variances between populations. A comparison of Figs. 4(a) and (b) confirms that overall change detection performance increases with $N$, and that, as expected with larger sample sizes, the crossover point for estimator performance tends toward unity. Since the number of samples that can be used in actual SAR coherent change detection applications is often quite limited, however, we can expect that the use of $|\hat{\rho}_b|$ in practical settings may be of overall benefit.

To test this hypothesis, we next formed a coherent change detection image from the SAR images shown in Fig. 1 using each of the two estimators $|\hat{\rho}_b|$ and $|\hat{\rho}_c|$, with $N = 5$ neighboring pixels chosen in a cross pattern. The results are shown in Fig. 5, from which it can be seen that the use of $|\hat{\rho}_b|$ results in a CCD image with comparatively higher overall contrast. A smaller $250 \times 250$ portion of Fig. 5 is shown in Fig. 6 to further demonstrate the difference between the two estimators in regions of nearly equal variance (here estimated as the ratio value $R = 0.9436$). Here again the higher contrast resulting from the use of $|\hat{\rho}_b|$ is apparent.

4. DISCUSSION AND FUTURE WORK

We conclude by noting that our investigation into the use of $|\hat{\rho}_b|$ in particular is motivated by the notion that in practice we can test for equal variance prior to deciding which estimator ($|\hat{\rho}_b|$ or $|\hat{\rho}_c|$) to employ. Specifically, the empirical changeover points observed here can be used to supply a threshold to the mean backscatter power ratio test analyzed in [5]. This is our main focus for future work.

In summary, here we have empirically investigated the detection-theoretic properties of two natural coherence magnitude estimators: a classical estimator $|\hat{\rho}_c|$ which does not assume equal population variances, and an estimator $|\hat{\rho}_b|$ which does. A direct comparison of distributional properties showed that when the equal-variance assumption holds and the true underlying coherence is equal to zero, then as expected $|\hat{\rho}_b|$ possesses better properties than $|\hat{\rho}_c|$. Furthermore, our empirical studies suggest that for small sample sizes, $|\hat{\rho}_b|$ can outperform $|\hat{\rho}_c|$ even when the true variances are far from being equal, in some relative sense. We noted that a decision between these two estimators can be made using local neighborhood data in the SAR imaging context, and suggested a two-step procedure to do so, which we are currently pursuing as a matter of future work.

5. REFERENCES