1. Interdisciplinary Ocean Science and Data Assimilation
2. Different Methods and their Applications
3. Error Subspace Statistical Estimation (ESSE)
   - Smoothing and Biogeochemical Dominant Dynamical Balances (Mass Bay/ Monterey Bay)
   - Error Forecasting, Adaptive Sampling and Adaptive Modeling in Monterey Bay
4. Conclusions
Processes/Platforms:
Time and Space Scales

Harvard University
P.F.J. Lermusiaux

OCEANIC FOOD WEB: Multiple trophic relations

e.g. leading to adult herring (arrows show energy flow)

• Interactions of Physical and Biological/Chemical Dynamical Processes, e.g.
  - Primary Productivity
  - The Biological Pump and its Role in the Changing Global Carbon Cycle
Physical and Multidisciplinary Observations

**AUV**
- WHOI Gliders
- SIO Gliders
- Dorado
- NPS REMUS
- Cal Poly REMUS

**Aircraft**
- Twin Otter
- P3 / AXBT / SST

**Ships**
- Pt. Sur
- John Martin

**Satellite**
- SST
- SeaWiFS

**Moored/Fixed**
- CODAR
- M1/M2
- NPS ADCP
- MBARI Profiler

**Drifting**
- Surface Drifters
- Profilers
Interdisciplinary Ocean Science and Data Assimilation

• From observations and a priori conservation laws, fundamental ocean science formulates models, usually differential equations, which aim to explain the dynamics of the sea phenomena under study

• Estimation of four-dimensional fields and parameters in the ocean is challenging
  – Multiple interactive scales in space and time (ocean weather: 1-100km, 1-10days)
  – Large domains (e.g. 10-1000km during 10-1000days)
  – Limited ocean data

• Coupled physical-biogeochemical-ecosystem-optical-acoustical modeling and estimations initiated

• Substantial advances require interdisciplinary data assimilation:
  – Quantitative combinations of data and models, in accord with uncertainties
  – Model reductions, simplifications and understanding
WHAT IS DATA ASSIMILATION?
A Melded Estimate of Data and Dynamics

Measurement Model
Measurement Errors

Data Assimilation Criterion

Dynamical Model
Model Errors

Melded Estimate (state, parameter, structure)
Agrees with measurement and dynamical models within respective uncertainties

A posteriori errors and uncertainties

Gulf of Cadiz, 1998

Real-Time HOPS/ESSE physical-ecosystem predictions
Generic Data Assimilation Problem

Dynamical models:

\[ d\phi_i + \mathbf{u} \cdot \nabla \phi_i \, dt - \nabla (K_i \nabla \phi_i) \, dt = B_i(\phi_1, \ldots, \phi_i, \ldots, \phi_n) \, dt + d\eta_i \quad (i = 1, \ldots, n) \]

\[ \text{e.g. } i = u, v, T, \ldots, ZOO, \ldots, p \]

Parameter equations:

\[ dP_\ell = C_\ell(\phi_1, \ldots, \phi_i, \ldots, \phi_n) \, dt + d\zeta_\ell \quad (\ell = 1, \ldots, p) \]

\[ \text{e.g. } P_\ell = \{ K_i, R_i, \ldots \} \]

Measurement models:

\[ y_j = H_j(\phi_1, \ldots, \phi_i, \ldots, \phi_n) + \epsilon_j \quad (j = 1, \ldots, m) \]

\[ \text{e.g. } y_j = \{ XBT_j, Fluo_j, SSH_j, CODAR_j \} \]

Assimilation criterion:

\[ \min_{\phi_i, P_\ell} \ J(d\eta_i, d\zeta_\ell, \epsilon_j, q_\eta, q_\zeta, q_\epsilon) \]
CLASSES OF DATA ASSIMILATION SCHEMES

• Estimation Theory (Filtering and Smoothing)
  1. Direct Insertion, Blending, Nudging - Linear
  2. Optimal interpolation - Linear
  3. Kalman filter/smooth - Linear
  4. Bayesian estimation (Fokker-Plank equations) - Non-lin.
  5. Ensemble/Monte-Carlo methods - Non-lin.
  6. Error-subspace/Reduced-order methods: Square-root filters, e.g. SEEK - (Non)-Lin.

• Control Theory/Calculus of Variations (Smoothing)
  1. “Adjoint methods” (+ descent) - Linear
  2. Generalized inverse (e.g. Representer method + descent) - Linear

• Optimization Theory (Direct local/global smoothing)
  1. Descent methods (Conjugate gradient, Quasi-Newton, etc) - Lin
  2. Simulated annealing, Genetic algorithms - Non-lin.

• Hybrid Schemes
  • Combinations of the above

Error Evol. | Criterion
-----------|---------
- Linear   | LS
- Linear   | LS
- Non-lin. | Non-LS
- Non-lin. | LS/Non-LS
- (Non)-Lin. | LS
- Non-lin. | LS/Non-LS
- Lin | LS/Non-LS
- Non-lin. | LS/Non-LS
Control Theory
(Calculus of Variation Approach, Variational Assimilation)

Adjoint Method

\[
\min_{\hat{\psi}} J_N = \epsilon_0^T P_0^{-1} \epsilon_0 + \sum_{k=1}^{N-1} v_k^T R_k^{-1} v_k + \sum_{k=1}^N 2\lambda_{k-1}^T w_{k-1}
\]  

(19)

Dynamical model \( \hat{\psi}_k = A_{k-1} \hat{\psi}_{k-1} \quad k = 1, \ldots, N \)  

(20)

Initial condition \( \hat{\psi}_0 = \Psi_0 + P_0 A_0^T \lambda_0 \)  

(21)

Adjoint model \( \lambda_{k-1} = A_k^T \lambda_k + H_k^T R_k^{-1} (y_k - H_k \hat{\psi}_k) \quad k = 1, \ldots, N - 1 \)  

(22)

Initial condition \( \lambda_{N-1} = 0 \)  

(23)
Georges Bank (NW Atlantic)

Estimate full biological source term (RHS) from data (Pseudicalanus spp.)


Figure 12.15 Top row: bimonthly climatological *Pseudocalanus* spp. distributions (adults only) objectively analyzed from the MARMAP data (number of animals m$^{-3}$). Second row: three source terms $B(x, y)$ resulting from three of the six inversions. Each $B(x, y)$ is located directly below the analyzed data used to initialize the experiment. That is, the JF–MA source term results in a forward model integration which matches the March–April analyzed data. Last two rows: two of the remaining terms in the ADR equation, adective flux divergence and overall tendency, averaged over the period of integration. Fields in the bottom three rows have been normalized to the bottom depth, so the units are number of animals m$^{-4}$s$^{-1}$. From D. J. McGillicuddy, Jr. et al. (1998)
Direct Minimization Methods
(descent methods, simulated annealing, genetic algorithms, etc)

Comparisons of methods for the estimation of biogeochemical parameters
Vallino, JMR (2000)

Figure 12.8 Optimized, scaled (0 to 1) parameter values associated with each of the minima located by the twelve optimization routines (abbreviated routine names and their symbols are on top of the figure). Model parameters (left ordinate) and their absolute parameter bounds (right ordinate) are described in Vallino (2000). Parameters marked with an asterisk (left ordinate) were held constant during the data assimilation. From Vallino (2000).
Stochastic and Hybrid Methods

Stochastic Methods
Based on (nonlinear) stochastic optimal control
Try to solve the conditional probability density equation (Fokker-Planck)
Minimum error variance, maximum likelihood or minimax estimates
    Monte Carlo ensemble calculations

Hybrid methods
Combinations of schemes, for both state and parameter estimation: e.g.,
    - Error Subspace Statistical Estimation (ESSE) schemes
    - Kalman filter first, then Representer method.
Miller et al, Tellus (1999)

Data assimilation into nonlinear stochastic models

Fig. 8. Results of data assimilation experiments. State variable $X$, reference solution, observations, and output of data assimilation scheme. (a) The extended Kalman filter (EKF). (b) The ensemble EKF. (c) The mode of the conditional distribution, calculated by Monte-Carlo and Bayes’ theorem.
Data Assimilation via ESSE

Table 1. Filtering/Smoothing via ESSE: Continuous-Discrete Problem Statement

<table>
<thead>
<tr>
<th>Dynamical Model:</th>
<th>(d\tilde{x} = \mathcal{M}(\tilde{x}) , dt + d\tilde{\eta}, ) with (\tilde{x}(r_0, t_0) = \tilde{x}_0 + \tilde{\eta}(0)).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement Model:</td>
<td>(y^\circ_k = \mathcal{H}(x_k) + \epsilon_k).</td>
</tr>
<tr>
<td>Estimation Criterion:</td>
<td>Estimate Error Subspace: (\left{\text{Find } P^p_k = E_k P_k E_k^T \text{ with } \text{rank}(E_k) = p \mid \min_{P_k, E_k} |P_k - P^p_k|\right})</td>
</tr>
<tr>
<td>Estimate State by</td>
<td>Min. Err. Var. in ES: (\left{\text{Find } \tilde{x}<em>k \mid \min</em>{x_k} J_k = \text{tr} [P^p_k(\cdot)] \text{ using } [y^\circ_0, ..., y^\circ_k/y^\circ_N]\right})</td>
</tr>
</tbody>
</table>

○ Optimal error space reduction and Min. Err. Var. combined:

“Estimate the ocean evolution by minimizing the largest (most energetic) expected errors, in agreement with the full dynamical model and measurement model (data) constraints, and their respective uncertainties.”

○ Linked to POD/Polynomial Chaos, but with time-varying error KL basis: \(x(x, t, \theta) = \bar{x}(x, t) + \sum_{i=1}^{M} \sqrt{\lambda_i} \, \phi^s_i(x, t) \, \zeta_i(\theta)\)
Error Subspace Statistical Estimation (ESSE)

- Uncertainty forecasts (with dynamic error subspace, error learning)
- Ensemble-based (with nonlinear and stochastic model)
- Multivariate, non-homogeneous and non-isotropic DA
- Consistent DA and adaptive sampling schemes
- Software: not tied to any model, but specifics currently tailored to HOPS
Data-Forecast Melding:
Minimum Error Variance within Error Subspace

TRUNCATED Minimum Sample ES Variance, Linear Update (subscript k omitted)

Dynamical State Update: \( \hat{x}(+) = \hat{x}(-) + K^p \left( y^o - \mathcal{H}(\hat{x}(-)) \right) \).

Sample ES Optimal Gain: \( K^p = E_- \Pi(-) H^p T \left( H^p \Pi(-) H^p T + R \right)^{-1} \), where \( H^p \doteq HE_- \).

Sample ES Cov. Update: \( L \Pi(+) L^T = \Pi(-) - \Pi(-) H^p T \left( H^p \Pi(-) H^p T + R \right)^{-1} H^p \Pi(-) \).
\[ E_+ = E_- L \, . \]

ADAPTIVE LEARNING of the Error Subspace (subscript k omitted)

\[ \hat{n}(+) = K_{trc} \left( y^o - \mathcal{H}(\hat{x}(+)) \right) \, , \]
\[ K_{trc} = E_{trc}(-) \Pi_{trc}(-) H_{trc}^T \left( H_{trc} \Pi_{trc}(-) H_{trc}^T + R \right)^{-1} \), where \( H_{trc} \doteq HE_{trc}(-) \).
\[ E_+^a \Sigma^a(+) V_+^a T = \text{SVD}_{p+1} \left( [E_+ \Sigma(+) , \hat{n}(+)] \right) \, , \]
\[ \Pi^a (+) = \frac{1}{q+1} \Sigma^a(+) \, . \]
Ocean Regions and Experiments/Operations for which ESSE has been utilized in real-time

- Strait of Sicily (AIS96-RR96), Summer 1996
- Ionian Sea (RR97), Fall 1997
- Gulf of Cadiz (RR98), Spring 1998
- Massachusetts Bay (LOOPS), Fall 1998
- Georges Bank (AFMIS), Spring 2000
- Massachusetts Bay (ASCOT-01), Spring 2001
- Monterey Bay (AOSN-2), Summer 2003

For publications, email me or see http://www.deas.harvard.edu/~pierrel
Massachusetts Bay

Horizontal Circulation Patterns for stratified conditions (not present at all times) and Coupled bio-physical sub-regions in late summer (Dominant dynamics for trophic enrichment and accumulation)

**Boston Harbor:** Charles River, sediments, toxic material, NO$_3$–NH$_4$
Along Coast: upwelling/downwelling $\Rightarrow$ bio $\uparrow/\downarrow$

**Open Bay:** submesoscale/mesoscale eddies. Ageostrophic $w \Rightarrow$ bio

**Cape Cod Bay:** Horizontal bio advection and submesoscales

**West of Stellwagen Bank:** GOM meanders, tides, topographic upwell/downwell

**Offshore:** GOM meanders

**Race Point:** Multiple bio advections, accumulation, and tides

**Cape Ann:** Physical instabilities at GOM inflow

Harvard University
Coupled Physical-Biogeochemical **Smoothing** via ESSE

Cross-sections in Chl-a fields, from south to north along main axis of Massachusetts Bay, with:

- **a) Nowcast on Aug. 25**
- **b) Forecast for Sep. 2**
- **c) 2D objective analysis for Sep. 2 of Chl-a data collected on Sep. 2–3**
- **d) ESSE filtering estimate on Sep. 2**
Coupled Physical-Biogeochemical DA via ESSE (continued)

e) Difference between ESSE smoothing estimate on Aug. 25 and nowcast on Aug. 25

f) Forecast for Sep. 2, starting from ESSE smoothing estimate on Aug. 25

(g): as d), but for Chl-a at 20 m depth

(h): RMS differences between Chl-a data on Sep. 2 and the field estimates at these data-points as a function of depth (specifically, “RMS-error” for persistence, dynamical forecast and ESSE filtering estimate)

Internal predictability: 2 weeks
Interactive Visualization and Targeting of pdfs

Advanced Visualization and Interactive Systems Lab: A. Pang, A. Love, W. Shen
A Quest for Dominant Dynamical Balances

- Ocean dynamics is complex, with multiple scales, processes and features
- Ultimate basic understanding is relatively simple but hard to reach
- Modern approach:
  - Combine data and dynamical models quantitatively for realistic studies
  - A road towards understanding and simplified dynamics
- Many oceanic features can be described by limited number terms, said to be in approximate ``balance’’: e.g. geostrophy, Ekman layer
- Focus here: explore dominant (dynamical) biogeochemical and biogeochemical-physical balances in coastal ecosystems
- Such balances are essential constraints for optimal sampling, biogeochemical initialization and selection of model parameters
COUPLED PHYSICAL-BIOGEOCHEMICAL MODELS

• Physical model: Primitive-Equation (PDE, x, y, z, t: HOPS)

Horiz. Mom. \( \frac{Du_h}{Dt} + f e_3 \wedge u_h = -\frac{1}{\rho_0} \nabla_h p_w + \nabla_h \cdot (A_h \nabla_h u_h) + \frac{\partial A_v \partial u_h}{\partial z} \) (1-2)

Vert. Mom. \( \rho g + \frac{\partial p_w}{\partial z} = 0 \) (3)

Thermal en. \( \frac{DT}{Dt} = \nabla_h \cdot (K_h \nabla_h T) + \frac{\partial K_v \partial T}{\partial z} \) (4)

Cons. of salt \( \frac{DS}{Dt} = \nabla_h \cdot (K_h \nabla_h S) + \frac{\partial K_v \partial S}{\partial z} \) (5)

Cons. of mass \( \nabla \cdot u = 0 \) (6)

Eqn. of state \( \rho(r, z, t) = \rho(T, S, p_w) \) (7)

• Biogeochemical model: Generic ADR equation (PDE, x, y, z, t)

\[ \frac{\partial \phi_i}{\partial t} + u \cdot \nabla \phi_i - \nabla_h (A_i \nabla_h \phi_i) - \frac{\partial K_i \partial \phi_i}{\partial z} = B_i(\phi_1, \ldots, \phi_i, \ldots, \phi_7) \] (8 – 14)

\( i = \text{NO}_3, \text{P}_{\text{NO}_3}, \text{ZOO}, \text{NH}_4, \text{DET}, \text{CHL}, \text{P}_{\text{NH}_4} \)
Schematic representation of ecosystem model (seven state variables/compartments)
Dominant dynamical balances for initial biogeochemical fields/parameters

Circadian (daily) 0th order biological balance

Phyto eqn.:

\[
\frac{1}{T} \int_0^T V^{NO_3} + V^{NH_4} dt \simeq gZ + n_3P
\]

\[
V^{NH_4} = \frac{PP}{\theta^C_{Chl} k_{NH_4} + NH_4} P, \quad V^{NO_3} = \frac{PP}{\theta^C_{Chl} k_{NO_3} + NO_3} e^{-\psi^{NH_4}}P, \quad P_P = P_m(1 - e^{-\alpha\bar{E}/P_m})e^{-\beta\bar{E}/P_m}
\]

Optical model: \( E(x, y, z, t) = E(x, y, 0, t)e^{-(k_{w}z + k_{c}\int_0^z Chl dz)} \)

Zoo eqn.: Select non-zero root,

\[
0 \simeq (1 - \gamma_1 - \gamma_2)g - n_1 - n_2Z, \quad g = R_m(1 - e^{-\Lambda P})
\]

Nitrate eqn.:

\[
\frac{1}{T} \int_0^T V^{NO_3} dt \simeq k_N[NH_4]
\]

Ammonium eqn.:

\[
\frac{1}{T} \int_0^T V^{NH_4} dt \simeq -k_N[NH_4] + \gamma_1gZ + (1 - \epsilon_1)n_1Z + (1 - \epsilon_2)n_2Z^2 + k_DD
\]

Detritus eqn.: \( \int_0^H dz \) with either null \( D = 0 \) or well-mixed \( \frac{\partial D}{\partial z} = 0 \) surf./bot. BC

\[
\nu_D \frac{\partial D}{\partial z} + k_D D \simeq \gamma_2gZ + \epsilon_1n_1Z + \epsilon_2n_2Z^2 + n_3P + n_4P^2
\]

Chlorophyll: OA-ed from Fluo. For \( P \), assume fully-acclimated C:CHL

Balance subject to observed variables and parameters constraints
Daily Average of RHS Terms: Residuals vs. Order of Magnitude

Mass. Bay: Aug 21,

Monterey Bay: Aug 04,
REGIONAL FEATURES of Monterey Bay and California Current System and Real-time Modeling Domains (AOSN2, 4 Aug. – 3 Sep., 2003)

• Upwelling centers at Pt AN/ Pt Sur: Upwelled water advected equatorward and seaward
• Coastal current, eddies, squirts, filam., etc: Upwelling-induced jets and high (sub)-mesoscale var. in CTZ
• California Undercurrent (CUC): Poleward flow/jet, 10-100km offshore, 50-300m depth
• California Current (CC): Broad southward flow, 100-1350km offshore, 0-500m depth
ESSE Surface Temperature Error Standard Deviation Forecasts

Start of Upwelling

First Upwelling period

End of Relaxation

Second Upwelling period
Which sampling on Aug 26 optimally reduces uncertainties on Aug 27?

4 candidate tracks, overlaid on surface T fct for Aug 26

Best predicted relative error reduction: track 1

2-day ESSE fct

IC(nowcast) Forecast DA DA of each track
Aug 24 Aug 26 Aug 27

DA 1 ESSE for Track 1
DA 2 ESSE for Track 2
DA 3 ESSE for Track 3
DA 4 ESSE for Track 4
Real-time Adaptive Coupled Models

- Different Types of Adaptive Couplings:
  - Adaptive physical model drives multiple biological models (biology hypothesis testing)
  - Adaptive physical model and adaptive biological model proceed in parallel, with some independent adaptation
- Numerical Implementation
  - For performance and scientific reasons, both modes are being implemented using message passing for parallel execution
  - Mixed language programming (using C function pointers and wrappers for functional choices)
Generalized Adaptable Biological Model

Diagram showing the flow of nutrients and biological processes, including:
- Nurticication
- Uptake
- DOM extraction
- Mortality
- Grazing
- Predation
- Respiration
- Sloppy feeding
- Aggregation
- Resuspension
- Sinking
- Remineralization
- Nutrient release
- Sediments
A Priori Biological Model
Example: Use P data to select parameterizations of Z grazing

Table 1. Parameterization of grazing on multiple types of prey with passive selection (\(g_{\text{max}}\): maximum grazing rate; K: Half-saturation constant (but saturation constant in Eq. 1); \(P_0\) threshold below which grazing is zero; \(p_i\): preference coefficient; \(?\), \(a\), \(?\): constant).

<table>
<thead>
<tr>
<th>Function</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Rectilinear</td>
<td>Armstrong, 1994</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{p_i P_i}{K} ), for (R \leq K); (g_{\text{max}}, ) for (R &gt; K), (R = \sum_{i=1}^{n} p_i P_i)</td>
<td></td>
</tr>
<tr>
<td>(2) Ivlev function for each prey type:</td>
<td>Leonard et al., 1999</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} (1 - e^{-\alpha_i})), with (R = \sum_{i=1}^{n} p_i P_i)</td>
<td></td>
</tr>
<tr>
<td>(3) Ivlev function with interference between prey types:</td>
<td>Hofmann and Ambler, 1988</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \left(1 - e^{-a_i} \frac{P_i}{R}\right)), with (R = \sum_{i=1}^{n} p_i P_i)</td>
<td></td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{a_i N_i}{1 + \sum_{j=1}^{n} a_j \tau_j N_j})</td>
<td></td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{p_i P_i}{K + \sum_{j=1}^{n} p_j P_j})</td>
<td></td>
</tr>
<tr>
<td>(6) Threshold MM function:</td>
<td>Evans, 1988; Lancelot et al., 2000</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \left(\frac{R - P_i}{K + R - P_i}\right) \frac{p_i P_i}{R}), with (R = \sum_{i=1}^{n} p_i P_i)</td>
<td></td>
</tr>
<tr>
<td>(7) Modified MM function:</td>
<td>Verity, 1991; Fasham et al. (1999) and Tian et al. (2001)</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{p_i P_i}{1 + \sum_{j=1}^{n} p_j P_j})</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Parameterization of grazing on multiple types of prey with active switching selection (\(g_{\text{max}}\): maximum grazing rate; K: Half-saturation constant; \(P_0\) threshold below which grazing is zero; \(p_i\): preference coefficient; \(\alpha\), \(a\), \(\tau\): constant).

<table>
<thead>
<tr>
<th>Function</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Switching MM predation:</td>
<td>Fasham et al., 1990; Strom and Loukos, 1998; Pitchford and Brindley, 1999; Spitz et al., 2001</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{p_i P_i}{K \sum_{j=1}^{n} p_j P_j + \sum_{j=1}^{n} p_j P_j^\alpha})</td>
<td></td>
</tr>
<tr>
<td>(2) Mechanistic disc switching predation:</td>
<td>Chesson, 1983</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{b_i N_i^\alpha}{(1 + c_i N_i)(1 + \sum_{j=1}^{n} b_j h_j N_j^\alpha)})</td>
<td></td>
</tr>
<tr>
<td>(3) Generalized switching function:</td>
<td>Tansky, 1978; Teramoto, 1979; Matsuda et al., 1986</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \left(\frac{p_i P_i}{\sum_{j=1}^{n} (p_j P_j)^\alpha}\right)^\alpha)</td>
<td></td>
</tr>
<tr>
<td>(4) Generalized switching function:</td>
<td>Vance, 1978</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{(p_i P_i)^\alpha}{\sum_{j=1}^{n} (p_j P_j)^\alpha})</td>
<td></td>
</tr>
<tr>
<td>(5) Generalized switching MM function:</td>
<td>Gismervik and Andersen (1997)</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \left(\frac{p_i P_i}{1 + \sum_{j=1}^{n} (p_j P_j)^\alpha}\right)^\alpha)</td>
<td></td>
</tr>
<tr>
<td>(6) Generalized switching MM function:</td>
<td>This work</td>
</tr>
<tr>
<td>(g_i = g_{\text{max}} \frac{(p_i (P_i - P_{\text{max}}))^\alpha}{1 + \sum_{j=1}^{n} (p_j (P_j - P_{\text{max}}))^\alpha})</td>
<td></td>
</tr>
</tbody>
</table>
Towards automated quantitative model aggregation and simplification

A priori configuration of generalized model on Aug 11 during an upwelling event

NPZ configuration of generalized model on Aug 11 during same upwelling event

Dr. Rucheng Tian
CONCLUSIONS

• ESSE powerful nonlinear scheme for interdisciplinary estimation of oceanic state variables and error fields via multivariate physical-biogeochemical-ecosystem-acoustical data assimilation

• Entering a new era of fully interdisciplinary oceanic dynamical system science, combining models and data

• Multiple novel and challenging opportunities, for example:
  – Quantitative assimilation feedbacks, e.g. via Adaptive (Bayesian) estimation/learning
    • Adaptive modeling/system identification (optimal parameters, structures, state variables and multi-model combinations)
    • Adaptive sampling (optimal data type, quantity and time-space locations)
    • Adaptive model reductions and simplifications
  – Theory and applications of environmental ocean science
    • Dominant dynamical balances for fundamental understanding, and for weak constraints