Nonparametric Bayesian state estimation in nonlinear dynamic systems with α-stable measurement noise

Nouha Jaoua, Emmanuel Duflos, Philippe Vanheeghe
LAGIS FRE CNRS 3303
Ecole Centrale de Lille
59651 Villeneuve d'Ascq, France
[nouha.jaoua,emmanuel.duflos,philippe.vanheeghe]@ec-lille.fr

1 Introduction

In signal processing literature, noise's source are often assumed to be Gaussian. However, in many fields the conventional Gaussian assumption is inadequate and leads to the loss of accuracy and/or resolution. This is particularly the case of applications operating at high frequency (60GHz) such as the internet of objects. In such context, underlying signals exhibit impulsive nature and do not have second order moments. Therefore, it is important in such applications to consider more realistic statistical models taking into account their specific characteristics.

Over the last decades, the α-stable distribution has attracted the attention from the signal processing community due to its efficiency in modeling impulsive data. In this work, we are interested in Bayesian inference in the case of α-stable noise. In literature, only few works consider this issue due to the lack of an analytical expression of the probability density function of α-stable distributions. Furthermore, most of these studies deal only with unimodal and symmetric α-stable distributions.

In this poster, we address the problem of state estimation in nonlinear state-space models with time-varying α-stable measurement noise. The latter can be multimodal and asymmetric. A flexible Bayesian nonparametric model based on Dirichlet Process Mixture (DPM) is introduced to model the measurement noise as an infinite mixture of α-stable distributions. A particle filter (PF) is then implemented to perform the online estimation. Simulation results demonstrate the efficiency and the robustness of the proposed algorithm.

2 DPM measurement noise model

Let us consider a general dynamic system described by the following discrete time state-space model

$$\begin{cases} x_{t+1} = g_t(x_t, w_t) \\ y_t = h_t(x_t, v_t) \end{cases}$$

where t is the time index, x_t is the state variable, y_t is the measurement, g_t and h_t are respectively the state and the observation functions, w_t is the process noise and v_t is the measurement noise. Here, we assume that w_t is Gaussian with known mean μ_t^v and variance Σ_t^v and v_t is α-stable with time-varying parameters $v_t \sim \mathcal{S}_{\alpha_t}(\beta_t, \gamma_t, \mu_t)$.

We suppose that the measurement noise distribution is a DPM. The mixed pdf $f(.|\theta_t)$ is assumed to be an α-stable distribution denoted $\mathcal{S}_{\alpha_t}(\beta_t, \gamma_t, \mu_t)$. We suppose that the parameters α_t and β_t are uniformly distributed on their supports. The parameter μ_t is assumed to be Gaussian with mean m^μ and variance Σ^μ. For the parameter γ_t, an inverse Gamma distribution with shape a^γ and scale b^γ is chosen. We denote α the scale parameter of the DPM. The base distribution $\mathbb{G}_0$ can be defined as the product of parameters priors:

$$\mathbb{G}_0 \sim \mathcal{U}]0,2] \times \mathcal{U}[-1,1] \times \mathcal{IG}(a^\gamma, b^\gamma) \times \mathcal{N}(m^\mu, \Sigma^\mu)$$

Finally, we obtain the following DPM model for the measurement noise distribution [1]

$$\begin{aligned} &\mathbb{G}|\Phi \sim DP(\mathbb{G}_0, \alpha) \\ &\theta_t|\mathbb{G} \sim \mathbb{G} \\ &v_t|\theta_t \sim \mathcal{S}_{\alpha_t}(\beta_t, \gamma_t, \mu_t) \end{aligned}$$

where $\Phi = \{\alpha, a^\gamma, b^\gamma, m^\mu, \Sigma^\mu\}$ and $\theta_t = \{\alpha_t, \beta_t, \gamma_t, \mu_t\}$ denote respectively the set of hyperparameters and the latent variable giving at each time index t the values of the α-stable distribution parameters. We assume here that hyperparameters are pre-specified and fixed. This model can be expressed equivalently as $v_t \sim F(v_t)$ where the measurement noise distribution $F(v_t)$ can be written as follows:

$$F(v_t) = \int f(.|\theta_t) d\mathbb{G}(\theta_t)$$

So, $F(v_t)$ can be seen as a countable infinite mixture of α-stable distributions with unknown parameters, and the mixing distribution $\mathbb{G}$ is sampled from a DP.

3 Particle filter for joint state and noise parameters estimation

Our main target is to jointly estimate the mixing distribution $\mathbb{G}$, the state x_t and the latent variable θ_t. In practice, only the state variable x_t is of interest, $\mathbb{G}$ and θ_t are considered as nuisance parameters. Within a Bayesian framework, we need to compute the joint posterior pdf $p(x_{0:t}, \theta_{1:t}, \mathbb{G}|y_{0:t}, \Phi)$. Thanks to the Polya urn representation [2], the mixing distribution $\mathbb{G}$ can be marginalized out from this pdf. Consequentially, the inference is from now based on $p(x_{0:t}, \theta_{1:t}|y_{0:t}, \Phi)$. However, the latter is analytically intractable. Therefore, we propose to use particle filtering techniques in order to find an estimate of the required posterior pdf.

4 Simulations

Results illustrated in Fig. 1 are obtained applying the proposed scheme on the following nonlinear model:

$$\begin{cases} x_{t+1} = 0.5x_t + 25\dfrac{x_t}{1+x_t^2} + 8\cos(1.2(t+1)) + w_t \\ y_t = 10x_t + v_t \end{cases}$$

Curves in Fig. 1.(a) show the estimated signal as well as the true and the observed ones. Fig. 1.(b)

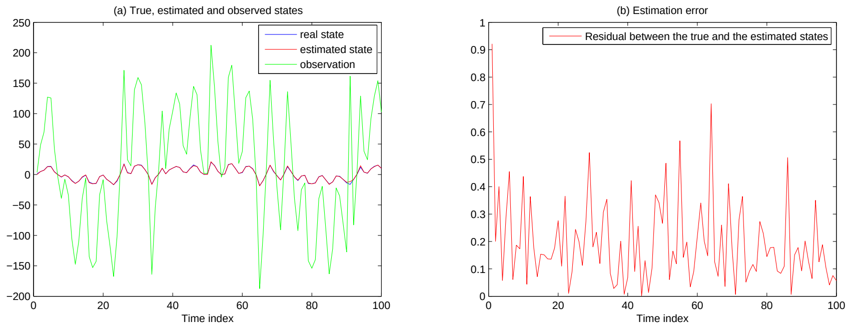

Figure 1: Estimation Results.

depicts the residual error between the true and the estimated states.

References

[1] Antoniak, C.E. (1974) Mixtures of Dirichlet Processes with Applications to Bayesian Nonparametric Problems *The Annals of Statistics* **2** (6):1152-1174.

[2] Blackwell, D. & Macqueen, J.B. (1973) Ferguson distributions via Polya urn schemes. *The Annals of Statistics* **1**:353-355.