AM 106/206: Applied Algebra **Prof. Salil Vadhan**

Lecture Notes 14

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1 Ideals

- Reading: Gallian Ch. 14
- Goal: ring-theoretic analogue of normal subgroup, something we can "mod out" (set to zero) to get a factor ring.
- Normal subgroups: since $a\varepsilon a^{-1} = \varepsilon$ in every group, we need $aNa^{-1} \subseteq N$ for N to work as an identity element in a factor group G/N.
- Ideals: since $a \cdot 0 = 0$ in every ring, we need $aI \subseteq I$ for I to work as an identity element in a factor ring R/I.
- Definition of ideal.
- Examples and Non-examples:
  - Ideals in $\mathbb{Z}$.
  - $R = \mathbb{R}[x]$, $I = \{p(x) : p(11) = 0\}$.
  - $R = \mathbb{R}[x]$, $I = \{p(x) : p(11) = 5\}$.
  - $R = \mathbb{R}[x]$, $I = \{a_2x^2 + \cdots + a_nx^n : n \geq 0, a_i \in \mathbb{R}\}$.
  - $R = \mathbb{C}[x]$, $I = \mathbb{Q}[x]$.
  - Ideals in a field.
  - Ideals in $\mathbb{Z}_n$.
  - Ideal $\langle a \rangle$ generated by $a \in R$.
  - Ideal $\langle a_1, \ldots, a_k \rangle$ generated by $a_1, \ldots, a_k \in R$.
  - $R = \mathbb{Z}$, $I = \langle m, n \rangle$.
  - $R = \mathbb{Q}[x]$, $I = \langle x - 7, x \rangle$.
  - $R = \mathbb{Z}[x]$, $I = \langle 17, x \rangle$.
- Theorem 14.2: Construction and Existence of Factor Ring R/I.