1 Ideals

- Reading: Gallian Ch. 14
- Goal: ring-theoretic analogue of normal subgroup, something we can “mod out” (set to zero) to get a factor ring.
- Normal subgroups: since $a \varepsilon a^{-1} = \varepsilon$ in every group, we need $aN a^{-1} \subseteq N$ for $N$ to work as an identity element in a factor group $G/N$.
- Ideals: since $a \cdot 0 = 0$ in every ring, we need $aI \subseteq I$ for $I$ to work as an identity element in a factor ring $R/I$.
- Definition of ideal.
- Examples and Non-examples:
  - Ideals in $\mathbb{Z}$.
  - $R = \mathbb{R}[x], I = \{p(x) : p(11) = 0\}$.
  - $R = \mathbb{R}[x], I = \{p(x) : p(11) = 5\}$.
  - $R = \mathbb{R}[x], I = \{a_2 x^2 + \cdots + a_n x^n : n \geq 0, a_i \in \mathbb{R}\}$.
  - $R = \mathbb{C}[x], I = \mathbb{Q}[x]$.
  - Ideals in a field.
  - Ideals in $\mathbb{Z}_n$.
  - Ideal $(a)$ generated by $a \in R$.
  - Ideal $(a_1, \ldots, a_k)$ generated by $a_1, \ldots, a_k \in R$.
  - $R = \mathbb{Z}, I = \langle m, n \rangle$.
  - $R = \mathbb{Q}[x], I = \langle x - 7, x \rangle$.
  - $R = \mathbb{Z}[x], I = \langle 17, x \rangle$.