

Harvard University
Computer Science 121

Problem Set 3

Due Tuesday, October 2, 2012 at 11:59 PM.

Submit your solutions electronically on the course website, located at <http://people.seas.harvard.edu/~salil/cs121/fall12/>. On the site, click the "Problem Set Submission" button and provide your login info. Once logged in, place the solutions to Parts A, B, and C in separate files named lastname-ps3a.pdf, lastname-ps3b.pdf, and lastname-ps3c.pdf respectively, in the appropriate dropboxes.

Late problem sets may be turned in until Friday, October 5, 2012 at 11:59 PM with a 20% penalty.

Problem set by ****ENTER YOUR NAME HERE****

Collaboration Statement: ****FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)****

See syllabus for collaboration policy.

Unless specified otherwise, assume the alphabet $\Sigma = \{a, b\}$

PART A (Graded by Jiapeng)

PROBLEM 1 (4+4+4+4+4 points)

For each of the following languages, determine whether the language is regular or non-regular, and *prove* your answer.

(A) $\{a^n b a^n : n \geq 1\}$

(B) $\{a^{3n} b^{5m} : n, m \in \mathbb{N}\}$

(C) $\{a^n b^m : n \geq m\}$

(D) $\{a^{2^n} : n \in \mathbb{N}\}^*$

(E) $\{w \in \Sigma^* : w \text{ contains exactly one of the substrings 'alan', 'mathison', or 'turing'}\}$, where Σ is the English alphabet (in lower case).

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PART B (Graded by Thomas)

PROBLEM 2 (3+3+3+3+3+3 points)

Classify the following sets as finite (in which case state the cardinality), countably infinite, or uncountably infinite. Give a *brief* justification. (A complete proof is not necessary.)

- (A) $\{\emptyset\}$
- (B) The set of syntactically valid computer programs in the programming language Python.
- (C) $\mathbb{Q} \times \mathbb{Q}$ (where \mathbb{Q} is the set of rational numbers)
- (D) $\{R \subseteq \mathbb{N} : \text{all numbers in } R \text{ are prime}\}$
- (E) $\{(x_1, x_2, \dots) : \text{each } x_i \in \mathbb{N} \text{ and for all } i, x_i \leq x_{i+1}\}$
- (F) $\{(x_1, x_2, \dots) : \text{each } x_i \in \mathbb{N} \text{ and for all } i, x_i \geq x_{i+1}\}$

PROBLEM 3 (6 points)

Show that a DFA with n states accepts an infinite language if and only if it accepts some string of length at least n .

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PART C (Graded by Nick)

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PROBLEM 4 (6+6+2 points)

Consider the language $F = \{a^i b^j c^k : i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

(A) Show that F is not regular.

(B) Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the following conditions: if s is any string in F of length at least p , then s may be divided into three pieces, $s = xyz$, such that

- (i) for each $i \geq 0$, $xy^i z \in F$
- (ii) $|y| > 0$
- (iii) $|xy| \leq p$

(C) Explain why the results from parts (A) and (B) do not contradict the pumping lemma.

PROBLEM 5 (Challenge 2 points)

Let L be the language of prime numbers in binary. That is $L = \{10, 11, 101, 111, 1011, 1101, \dots\}$. Prove that L is not regular. (Hint: Fermat's little theorem states that, for any prime p and positive integer x that is not a multiple of p , $x^{p-1} \bmod p = 1$.)