

Harvard University Extension School
Computer Science E-207

Problem Set 7

Due Friday, November 9, 2012 at 11:59 PM Eastern Time.

Submit your solutions in a single PDF called lastname+ps7.pdf emailed to cscie207@seas.harvard.edu.

LATE PROBLEM SETS WILL NOT BE ACCEPTED.

Problem set by ****ENTER YOUR NAME HERE****

Collaboration Statement: ****FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)****

See syllabus for collaboration policy.

Note: For all proofs involving constructions of Turing Machines, please give a *high-level* description. For examples, see the lecture on October 30 or Sipser Chapter 4.1.

PROBLEM 1 (10 points)

Let $L = \{\langle M \rangle \mid M \text{ is a DFA and for every string } w, \text{ if } M \text{ accepts } w, \text{ then } M \text{ also accepts } w^R\}$. Show that L is decidable.

PROBLEM 2 (10 points)

Let $L = \{\langle M \rangle \mid M \text{ moves left on the tape at some point when run on } \varepsilon\}$. Show that L is decidable.

PROBLEM 3 (10 points)

Show that a language L_1 is Turing-recognizable if and only if there exists some decidable language L_2 such that $L_1 = \{x : \text{there exists } y \text{ such that } \langle x, y \rangle \in L_2\}$.

(Hint: Imagine that y gives you some information about an accepting computation on x , if one exists.)

PROBLEM 4 (8 points)

Show that a language L is decidable if and only if there is an enumerator that outputs the elements of L in lexicographic order.

PROBLEM 5 (5 points)

Show that the class of Turing-recognizable languages is closed under union.

PROBLEM 6 (2+2+5+(2) points)

A *general grammar* is like a CFG except that the production rules are not restricted to transforming one variable at a time without any context information. The left hand side of each rule can contain multiple variables and alphabet symbols (rather than just one variable), but there must be at least one variable on the left hand side. That is, the rules are of the form $w \rightarrow w'$ where $w, w' \in (\Sigma \cup V)^*$ and $w \notin \Sigma^*$. For example, consider the general grammar $G = (V, \Sigma, R, S)$ with $V = \{S, B, C\}$, $\Sigma = \{a, b, c\}$, and R containing the following rules:

$$S \rightarrow aSBC|\varepsilon,$$

$$CB \rightarrow BC,$$

$$aB \rightarrow ab,$$

$$bB \rightarrow bb,$$

$$bC \rightarrow bc,$$

$$cC \rightarrow cc$$

This yields the string $aabbcc$ as follows.

$$S \Rightarrow aSBC \Rightarrow aaSBCBC \Rightarrow aaBCBC \Rightarrow aaBBCC \Rightarrow aabBCC \Rightarrow aabbCC \Rightarrow aabbcC \Rightarrow aabbcc.$$

- (A) What language does the above example general grammar generate? Is this language context-free?
- (B) Give a formal definition for what it means for a general grammar G to generate a string $w \in \Sigma^*$.
- (C) Show that every language generated by a general grammar is Turing-recognizable. (Hint: Use nondeterministic Turing machines.)
- (D) (Challenge) Prove that every Turing-recognizable language can be generated by a general grammar. This, along with the previous part, implies that general grammars are equivalent to Turing-recognizable languages. (Hint: You may assume that, without loss of generality, a Turing machine only halts with an empty tape and the head at the beginning of the tape.)