CS208: Applied Privacy for Data Science
Implementing (Centralized) Differential Privacy:
One-Shot Releases

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March 8, 2019
Review: Doing Better than Composition

• Not all sequences of $k$ queries require error growing as $\sqrt{k}$.

• **Example:** histograms
  • Let $B_1, \ldots, B_k \subseteq \mathcal{X}$ be disjoint bins.
  • Define $q_j : \mathcal{X}^n \rightarrow \{0,1\}$ by $q_j(x) = \# \{i : x_i \in B_j\}$.
  • Define $M(x) = (q_1(x) + Z_1, q_2(x) + Z_2, \ldots, q_k(x) + Z_k)$ where the $Z_j$’s are independent $\text{Lap}(2/\varepsilon)$ or $\text{Geo}(2/\varepsilon)$.
  • Then $M$ is $\varepsilon$-DP.
Synthetic Data via DP Histograms

• Use singleton bins $B_y = \{y\}$ for each $y \in \mathcal{Y}$.
• Construct a DP histogram $(a_1, \ldots, a_{|\mathcal{X}|}) \leftarrow M_{\text{hist}}(x)$, where $a_y \approx \#\{i : x_i = y\}$.
• Output synthetic dataset $\hat{x}$ with $a_y$ copies of each element $y$.

Difficulties?
• $a_y$’s may not be nonnegative integers.
  • Soln 1: use Geometric Mechanism and clamp at 0.
  • Soln 2: use Exponential Mechanism with range $\{0, \ldots, n\}$.
• Poor utility & efficiency when $\mathcal{X}$ is large.
A Practical Method to Reduce Privacy Loss when Disclosing Statistics Based on Small Samples

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March 2019
Publishing Statistics Based on Small Cells

- Social scientists increasingly use confidential data to publish statistics based on cells with a small number of observations
  - Causal effects of schools or hospitals [e.g., Angrist et al. 2013, Hull 2018]
  - Local area statistics on health outcomes or income mobility [e.g., Cooper et al. 2015, Chetty et al. 2018]
Intergenerational Mobility in the United States
Mean Child Household Income Rank vs. Parent Household Income Rank

Predicted Value Given Parents at 25th Percentile = 41st Percentile
Parents at 25th Percentile = $31,900

Source: Chetty, Friedman, Hendren, Jones, Porter (2018)
Geography of Upward Mobility in the United States
Average Income at Age 35 for Children whose Parents Earned $25,000 (25th percentile)

Note: Blue = More Upward Mobility, Red = Less Upward Mobility
Source: Chetty, Friedman, Hendren, Jones, and Porter 2018
Problem with releasing such estimates at smaller geographies (e.g., Census tract): risk of disclosing an individual’s data

Literature on differential privacy has developed practical methods to protect privacy for simple statistics such as means and counts [Dwork 2006, Dwork et al. 2006]

But methods for disclosing more complex estimates, e.g. regression or quasi-experimental estimates, are not feasible for many social science applications [Dwork and Lei 2009, Smith 2011, Kifer et al. 2012]
This Paper: A Practical Method to Reduce Privacy Loss

- We develop and implement a simple method of controlling privacy loss when disclosing arbitrarily complex statistics in small samples
  - The “Maximum Observed Sensitivity” (MOS) algorithm

- Method outperforms widely used methods such as cell suppression both in terms of privacy loss and statistical accuracy
  - Does not offer a formal guarantee of privacy, but potential risks occur only at more aggregated levels (e.g., the state level)
Example Regression from One Small Cell

Source: Authors' simulations.

25th percentile predicted value $\theta = 0.212$
Maximum Observed Sensitivity

- Our method: use the maximum observed local sensitivity across all cells in the data
  - In geography of opportunity application, calculate local sensitivity in every tract
  - Then use the maximum observed sensitivity (MOS) across all tracts within a given state as the sensitivity parameter for every tract in that state

- Analogous to Empirical Bayes approach of using actual data to construct prior on possible realizations rather than considering all possible priors
Maximum Observed Sensitivity Envelope

Source: Authors’ simulations.

Maximum Observed Sensitivity Among Tracts with 50 People = 0.21
Computing Maximum Observed Sensitivity

$MOSE = \frac{\chi}{N} = \frac{13.02}{N}$

Source: Authors’ simulations.
Producing Noise-Infused Estimates for Public Release

- Use max observed sensitivity $\chi$, tract counts, and exogenously specified privacy parameter $\varepsilon$ to add noise and construct public estimates:

\[
\tilde{\theta}_g = \theta_g + L \left( 0, \frac{\chi}{\varepsilon N_g} \right) \\
\tilde{N}_g = N_g + L \left( 0, \frac{1}{\varepsilon} \right)
\]

- This method not “provably private,” but it reduces privacy risk to release of the single max observed sensitivity parameter ($\chi$)
  
  - Privacy loss from release of regression statistics themselves is controlled below risk tolerance threshold $\varepsilon$

- Critically, $\chi$ can be computed at a sufficiently aggregated level that disclosure risks are considered minimal ex-ante
  
  - Ex: Census Bureau currently does not consider most statistics released at state or
Comparison to Alternative Methods: Statistical Bias

- In noise-infused data, regression provides an unbiased estimate of the (strong positive) relationship between teenage-birth rates for black women and single-parent share.
  - More generally, can adjust for noise using the “signal correlation”

- In contrast, count-based suppression generates bias that eliminates the result, since induces correlated measurement error from two sources:
  - Suppressing cells with few teenage births mechanically omits tracts with low teenage birth rates, which are concentrated in areas with few single parents.
  - Areas with a smaller black population (i.e., less diversity) have fewer teenage births and fewer areas with few single parents

- Identifying and correcting for these biases would be very difficult if one only had access to the post-suppression data
Comparison to Alternative Methods: Statistical Precision

- Primary concern of end users: will estimates be too noisy to be useful?
  - In Atlas, noise added to protect privacy was similar to inherent noise due to sampling error → estimates remain highly accurate
  - E.g., added privacy noise reduces reliability (i.e., fraction of total variance that is signal) only from 71.8% to 71.0%
Variance Decomposition for Tract-Level Estimates

Teenage Birth Rate For Black Women With Parents at 25th Percentile

- Total Variance
- Signal Variance
- Sampling Noise Variance
- Privacy Noise Variance

3% increase in noise var.

Source: Chetty, Friedman, Hendren, Jones, Porter (2018)
Conclusion

- Main lesson: tools from differential privacy literature can be adapted to control privacy loss while improving statistical inference

- Opportunity Atlas has been used by half a million people, by housing authorities to help families move to better neighborhoods, and in downstream research [Creating Moves to Opportunity Project; Morris et al. 2018]

- The MOS algorithm can be practically applied to any empirical estimate

- Example: difference-in-differences or regression discontinuity

- Even when there is only one quasi-experiment, pretend that a similar change occurred in other cells of the data and compute MOS across all cells
Future Work

- Two areas for further work that could increase use of differential privacy methods in social science:
  
  1. Developing formal metrics for risk of privacy loss for algorithms in which a single statistic (e.g., sensitivity) is released at a broader level of aggregation
  
  2. Developing techniques that can be applied to many estimators without requiring users to develop new algorithms for each application