Time-Varying Causal Treatment Effects

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Outline

• Introduction to mobile health

• Causal Treatment Effects (aka Causal Excursions)

• (A wonderfully simple) Estimation Method

• HeartSteps
HeartSteps

Context provided via data from:
- **Wearable band** → activity and sleep quality;
- **Smartphone sensors** → busyness of calendar, location, weather;
- **Self-report** → stress, user burden

In which contexts should the smartphone provide the user with a tailored activity suggestion?
Data from wearable devices that sense and provide treatments

- On each individual: \( O_1, A_1, Y_2, ..., O_t, A_t, Y_{t+1}, ... \)

- \( t \): Decision point
- \( O_t \): Observations at \( t^{th} \) decision point (high dimensional)
- \( A_t \): Treatment at \( t^{th} \) decision point (aka: action)
- \( Y_{t+1} \): Proximal outcome (aka: reward, utility, cost)
Structure of Mobile Health Intervention

1) Decision Points: times, $t$, at which a treatment might be provided.
   1) Regular intervals in time (e.g. every minute)
   2) At user demand

Heart Steps: approximately every 2-2.5 hours: pre-morning commute, mid-day, mid-afternoon, evening commute, after dinner
Structure of Mobile Health Intervention

2) Observations $O_t$
   1) Passively collected (via sensors)
   2) Actively collected (via self-report)

Heart Steps: classifications of activity, location, weather, step count, busyness of calendar, user burden, adherence…….
Structure of Mobile Health Intervention

3) Treatment $A_t$
   1) Types of treatments/engagement strategies that can be provided at a decision point, $t$
   2) Whether to provide a treatment

HeartSteps: tailored activity suggestion (yes/no)
Availability

• Treatments, $A_t$, can only be delivered at a decision point if an individual is available.  
  – $O_t$ includes $I_t=1$ if available, $I_t=0$ if not

• Treatment effects at a decision point are conditional on availability.

• Availability is not the same as adherence!
4) Proximal Outcome $Y_{t+1}$

Heart Steps: Step count over 30 minutes following decision point, $t$
Continually Learning Mobile Health Intervention

1) Trial Designs: Are there effects of the actions on the proximal response? *experimental design*

2) Data Analytics for use with trial data: Do effects vary by the user’s internal/external context,? Are there delayed effects of the actions? *causal inference*

3) Learning Algorithms for use with trial data: Construct a “warm-start” treatment policy. *batch Reinforcement Learning*

4) Online Algorithms that personalize and continually update the mHealth Intervention. *online Reinforcement Learning*
Micro-Randomized Trial Data

On each of $n$ participants and at each of $t=1,\ldots,T$ decision points:

- $O_t$ observations at decision point $t$,
  - includes $I_t=1$ if available, $I_t=0$ if not
- $A_t=1$ if treated, $A_t=0$ if not treated at decision $t$
  - Randomized, $p_t(H_t) = P[A_t=1| H_t, I_t=1]$
- $Y_{t+1}$ proximal outcome

$H_t=\{(O_i, A_i, Y_{i+1}), i=1,\ldots,t-1; O_t\}$ denotes data through $t$
Conceptual Models

Generally data analysts fit a series of increasingly more complex models:

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

and then next,

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

and so on…

- \( Y_{t+1} \) is activity over 30 min. following \( t \)
- \( A_t = 1 \) if activity suggestion and 0 otherwise
- \( Z_t \) summaries formed from \( t \) and past/present observations
- \( S_t \) potential moderator (e.g., current weather is good or not)
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and so on…

\( \alpha_1^T Z_t \) is used to reduce the noise variance in \( Y_{t+1} \)

\((Z_t\ is\ sometimes\ called\ a\ vector\ of\ control\ variables)
Causal, Marginal Effects

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

\( \beta_0 \) is the effect, marginal over all observed and all unobserved variables, of the activity suggestion on subsequent activity.

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

\( \beta_0 + \beta_1 \) is the effect when the weather is good (\( S_t = 1 \)), marginal over other observed and all unobserved variables, of the activity suggestion on subsequent activity.
Goal

• Develop data analytic methods that are consistent with the scientific understanding of the meaning of the $\beta$ coefficients

• Challenges:
  • Time-varying treatment ($A_t, t=1,...T$)
  • “Independent” variables: $Z_t, S_t, I_t$ that may be affected by prior treatment

• Robustly facilitate noise reduction via use of controls, $Z_t$
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Potential Outcomes

- $\bar{A}_t = \{A_1, A_2, \ldots, A_t\}$ (random treatments),
  $\bar{a}_t = \{a_1, a_2, \ldots, a_t\}$ (realizations of treatments)

- $Y_{t+1}(\bar{a}_t)$ is one potential proximal response

- $I_t(\bar{a}_{t-1})$ is one potential “available for treatment” indicator

- $H_t(\bar{a}_{t-1})$ is one potential history vector
  - $S_t(\bar{a}_{t-1})$ is a vector of features of history $H_t(\bar{a}_{t-1})$
Marginal & Causal Effect

Excursion effect at decision point $t$:

$$E[Y_{t+1}(\tilde{A}_{t-1}, 1) - Y_{t+1}(\tilde{A}_{t-1}, 0) \mid I_t(\tilde{A}_{t-1}) = 1, S_t(\tilde{A}_{t-1})]$$

- Effect is conditional on availability; only concerns the subpopulation of individuals available at decision $t$

- Effect is marginal over any $Y_u$, $u \leq t$, $A_u, u < t$ not in $S_t(\tilde{A}_{t-1})$---over all variables not in $S_t(\tilde{A}_{t-1})$. 
Excursion effect at decision point $t$:

$$E[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) | I_t(\bar{A}_{t-1}) = 1, S_t(\bar{A}_{t-1})]$$

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Consistency & Micro-Randomized $A_t \rightarrow$

\[
E[Y_{t+1}(\tilde{A}_{t-1}, 1) - Y_{t+1}(\tilde{A}_{t-1}, 0) \mid I_t(\tilde{A}_{t-1}) = 1, S_t(\tilde{A}_{t-1})]
\]

\[
= E\left[ E[Y_{t+1} \mid A_t = 1, I_t = 1, H_t] - E[Y_{t+1} \mid A_t = 0, I_t = 1, H_t] \mid I_t = 1, S_t \right]
\]

\[
= E\left[ \frac{A_t Y_{t+1}}{p_t(H_t)} - \frac{(1 - A_t) Y_{t+1}}{(1 - p_t(H_t))} \mid I_t = 1, S_t \right]
\]

($p_t(H_t)$ is randomization probability)
Marginal Treatment Effect

Treatment Effect Model:

\[
E[ E[Y_{t+1}|A_t = 1, I_t = 1, H_t] \\
- E[Y_{t+1}|A_t = 0, I_t = 1, H_t]| I_t = 1, S_t] \\
= S_t^T \beta
\]

\(H_t\) is participant’s data up to and at time \(t\)

\(S_t\) is a vector of data summaries and time, \(t\), \((S_t \subseteq H_t)\)

\(I_t\) indicator of availability

We aim to conduct inference about \(\beta\)!
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“Centered and Weighted Least Squares Estimation”

• Simple method for complex data
• Enables unbiased inference for a causal, marginal, treatment effect (the $\beta$’s)
• Inference for treatment effect is not biased by how we use the controls, $Z_t$, to reduce the noise variance in $Y_{t+1}$

https://arxiv.org/abs/1601.00237
Estimation

- Select probabilities: \( \tilde{p}_t(s) \in (0,1) \)
- Form weights: \( W_t = \left( \frac{\tilde{p}_t(S_t)}{p_t(H_t)} \right)^{A_t} \left( \frac{1-\tilde{p}_t(S_t)}{1-p_t(H_t)} \right)^{1-A_t} \)
- Center treatment actions: \( A_t \rightarrow (A_t - \tilde{p}_t(S_t)) \)
- Minimize:
  \[
  E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t)) S_t^T \beta)^2 I_t W_t \right]
  \]
- \( E_n \) is empirical distribution over individuals.
Minimize

\[ E_n \left[ \sum_{t=1}^{T} \left( Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta \right)^2 I_t W_t \right] \]

Good but incorrect intuition:

Appears to be a weighted “GEE” with a working independence correlation matrix and a centered treatment indicator, \( A_t - \tilde{p}_t(S_t) \) thus:

- \( E[Y_{t+1} | A_t, I_t = 1, Z_t] = Z_t^T \alpha + (A_t - \tilde{p}_t(S_t))S_t^T \beta \)

\( E_n \) is expectation with respect to empirical distribution.
Minimize

\[ E_n \left\{ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta)^2 I_t W_t \right\} \]

Good but incorrect intuition:

- \( E[Y_{t+1} | A_t, I_t = 1, Z_t] \neq Z_t^T \alpha + (A_t - \tilde{\rho}_t(S_t))S_t^T \beta \)

\( E_n \) is expectation with respect to empirical distribution.
Minimize

\[ E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta)^2 I_t W_t \right] \]

**The Modeling Assumption:**

\[ E[(E[Y_{t+1}|A_t = 1, I_t = 1, H_t] - E[Y_{t+1}|A_t = 0, I_t = 1, H_t])|I_t = 1, S_t] = S_t^T \beta_0 \]

If \( \tilde{\rho}_t \) depends at most on features in \( S_t \), then, under moment conditions, \( \hat{\beta} \) is consistent for \( \beta_0 \)
Theory

Under moment conditions, $\sqrt{n}(\hat{\beta} - \beta_0)$ converges to a Normal distribution with mean $0$ and var-covar matrix, $(\Sigma_p)^{-1}\Sigma(\Sigma_p)^{-1}$

$$\Sigma_p = E\left[\sum_{t=1}^{T} \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_tS_tS_t^T\right]$$

$Z_t$ and $S_t$ are finite dimensional feature vectors.
Gains from Randomization

• Causal inference for a marginal treatment effect
• Inference on treatment effect is robust to working model:

\[
E[Y_{t+1} \mid I_t = 1, H_t] \approx Z_t^T \alpha
\]

- \(Z_t \subseteq H_t\)
- Contrast to literature on partially linear, single index models and varying coefficient models
Price due to Marginal Estimand

This “GEE-like” method can only use a working independence correlation matrix

– Estimating function is biased if off-diagonal elements in working correlation matrix: in general,

\[
E \left[ (Y_{t+1} - Z_t^T \alpha \\
- (A_t - \tilde{p}_t(S_t))S_t^T \beta) I_t W_t I_u W_u \sigma_{t,u} Z_u \right] \neq 0
\]

if \( u \neq t \)
Choice of Weights

Choice of $\tilde{\rho}_t(S_t)$ determines marginalization over time under model misspecification of treatment effect.

Example: $S_t = 1$, $\tilde{\rho}_t(S_t) = \tilde{\rho}$. Resulting $\hat{\beta}$ is an estimator of

$$\sum_{t=1}^{T} E[I_t] \beta_t \bigg/ \sum_{t=1}^{T} E[I_t]$$

where

$$\beta_t = E[ E[Y_{t+1} | A_t = 1, I_t = 1, H_t]$$

$$- E[Y_{t+1} | A_t = 0, I_t = 1, H_t] | I_t = 1 ]$$
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Heart Steps Pilot Study

On each of $n=37$ participants:

a) Activity suggestion, $A_t$
   
   • **Provide a suggestion with probability .6**
     
     • a tailored sedentary-reducing activity suggestion (probability=.3)
     
     • a tailored walking activity suggestion (probability=.3)
   
   • **Do nothing (probability=.4)**

• 5 times per day * 42 days= 210 decision points
Conceptual Models

\[ Y_{t+1} \sim a_0 + a_1 Z_t + \beta_0 A_t \]
\[ Y_{t+1} \sim a_0 + a_1 Z_t + a_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

- \( t=1, \ldots T=210 \)
- \( Y_{t+1} \) = log-transformed step count in the 30 minutes after the \( t^{th} \) decision point,
- \( A_t = 1 \) if an activity suggestion is delivered at the \( t^{th} \) decision point; \( A_t = 0 \), otherwise,
- \( Z_t \) = log-transformed step count in the 30 minutes prior to the \( t^{th} \) decision point,
- \( d_t \) = days in study; takes values in \((0,1,\ldots,41)\)
Pilot Study Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t, \] and

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \alpha_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

<table>
<thead>
<tr>
<th>Causal Effect Term</th>
<th>Estimate</th>
<th>95% CI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 A_t ) (effect of an activity suggestion)</td>
<td>( \hat{\beta}_0 = .13 )</td>
<td>(-0.01, 0.27)</td>
<td>.06</td>
</tr>
<tr>
<td>( \beta_0 A_t + \beta_1 A_t d_t ) (time trend in effect of an activity suggestion)</td>
<td>( \hat{\beta}_0 = .51 )</td>
<td>(.20, .81)</td>
<td>&lt;.01</td>
</tr>
<tr>
<td></td>
<td>( \hat{\beta}_1 = -.02 )</td>
<td>(-.03, -.01)</td>
<td>&lt;.01</td>
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</table>
Heart Steps Pilot Study

On each of $n=37$ participants:

a) Activity suggestion

- Provide a suggestion with probability .6
  - a tailored walking activity suggestion (probability=.3)
  - a tailored sedentary-reducing activity suggestion (probability=.3)
- Do nothing (probability=.4)

- 5 times per day * 42 days = 210 decision points
Pilot Study Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_{1t} + \beta_1 A_{2t} \]

- \( A_{1t} = 1 \) if walking activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{1t} = 0 \), otherwise,
- \( A_{2t} = 1 \) if sedentary-reducing activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{2t} = 0 \), otherwise,

<table>
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<tbody>
<tr>
<td>( \beta_0 A_{1t} + \beta_1 A_{2t} )</td>
<td>( \hat{\beta}_0 = .21 ) ( \hat{\beta}_1 &gt;0 )</td>
<td>(.04, .39) ns</td>
<td>.02 ns</td>
</tr>
</tbody>
</table>
Initial Conclusions

• The data indicates that there is a causal effect of the activity suggestion on step count in the succeeding 30 minutes.
  • This effect is primarily due to the walking activity suggestions.
  • This effect deteriorates with time
  • The walking activity suggestion initially increases step count over succeeding 30 minutes by $\approx 271$ steps but by day 21 this increase is only $\approx 65$ steps.
Discussion

Problematic Analyses

• GLM & GEE analyses
• Random effects models & analyses
• Machine Learning Generalizations:
  – Partially linear, single index models & analysis
  – Varying coefficient models & analysis

--These analyses do not take advantage of the micro-randomization. Can accidentally eliminate the advantages of randomization for estimating causal effects--
Discussion

• Randomization enhances:
  – Causal inference based on minimal structural assumptions

• Challenge:
  – How to include random effects which reflect scientific understanding (“person-specific” effects) yet not destroy causal inference?
Collaborators!
Minimize

\[ E_n \left[ \sum_{t=1}^{T} \left( Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta \right)^2 I_t W_t \right] \]

- Resulting \( \hat{\beta} \) is an estimator of

\[ \left( E \sum_{t=1}^{T} \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_t S_t S_t^T \right)^{-1} E \left[ \sum_{t=1}^{T} S_t \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_t \beta_t(S_t) \right] \]