Time-Varying Causal Treatment Effects

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Outline

• Introduction to mobile health

• Causal Treatment Effects (aka Causal Excursions)

• (A wonderfully simple) Estimation Method

• HeartSteps
HeartSteps

Context provided via data from:
- **Wearable band** → activity and sleep quality;
- **Smartphone sensors** → busyness of calendar, location, weather;
- **Self-report** → stress, user burden

In which contexts should the smartphone provide the user with a tailored activity suggestion?
Data from wearable devices that sense and provide treatments

- On each individual: $O_1, A_1, Y_2, ..., O_t, A_t, Y_{t+1}, ...$

- $t$: Decision point

- $O_t$: Observations at $t^{th}$ decision point (high dimensional)

- $A_t$: Treatment at $t^{th}$ decision point (aka: action)

- $Y_{t+1}$: Proximal outcome (aka: reward, utility, cost)
Structure of Mobile Health Intervention

1) Decision Points: times, $t$, at which a treatment might be provided.
   1) Regular intervals in time (e.g. every minute)
   2) At user demand

Heart Steps: approximately every 2-2.5 hours: pre-morning commute, mid-day, mid-afternoon, evening commute, after dinner
Structure of Mobile Health Intervention

2) Observations $O_t$
   1) Passively collected (via sensors)
   2) Actively collected (via self-report)

HeartSteps: classifications of activity, location, weather, step count, busyness of calendar, user burden, adherence……..
Structure of Mobile Health Intervention

3) Treatment $A_t$
   1) Types of treatments/engagement strategies that can be provided at a decision point, $t$
   2) Whether to provide a treatment

HeartSteps: tailored activity suggestion (yes/no)
Availability

- Treatments, $A_t$, can only be delivered at a decision point if an individual is available.
  - $O_t$ includes $I_t=1$ if available, $I_t=0$ if not

- Treatment effects at a decision point are conditional on availability.

- Availability is not the same as adherence!
4) Proximal Outcome $Y_{t+1}$

**Heart Steps**: Step count over 30 minutes following decision point, $t$
Continually Learning Mobile Health Intervention

1) Trial Designs: Are there effects of the actions on the proximal response? *experimental design*

2) Data Analytics for use with trial data: Do effects vary by the user’s internal/external context? Are there delayed effects of the actions? *causal inference*

3) Learning Algorithms for use with trial data: Construct a “warm-start” treatment policy. *batch Reinforcement Learning*

4) Online Algorithms that personalize and continually update the mHealth Intervention. *online Reinforcement Learning*
Micro-Randomized Trial Data

On each of $n$ participants and at each of $t=1,..,T$ decision points:

- $O_t$ observations at decision point $t$,
  - includes $I_t=1$ if available, $I_t=0$ if not
- $A_t=1$ if treated, $A_t=0$ if not treated at decision $t$
  - Randomized, $p_t(H_t) = P[A_t=1 \mid H_t, I_t=1]$
- $Y_{t+1}$ proximal outcome

$H_t = \{(O_i, A_i, Y_{i+1}), i=1,...,t-1; O_i\}$ denotes data through $t$
Conceptual Models

Generally data analysts fit a series of increasingly more complex models:

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

and then next,

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

and so on…

- \( Y_{t+1} \) is activity over 30 min. following \( t \)
- \( A_t = 1 \) if activity suggestion and 0 otherwise
- \( Z_t \) summaries formed from \( t \) and past/present observations
- \( S_t \) potential moderator (e.g., current weather is good or not)
Conceptual Models

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and so on…

\( \alpha_1^T Z_t \) is used to reduce the noise variance in \( Y_{t+1} \)

\( Z_t \) is sometimes called a vector of control variables.
Causal, Marginal Effects

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

\( \beta_0 \) is the effect, marginal over all observed and all unobserved variables, of the activity suggestion on subsequent activity.

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

\( \beta_0 + \beta_1 \) is the effect when the weather is good \((S_t=1)\), marginal over other observed and all unobserved variables, of the activity suggestion on subsequent activity.
Goal

• Develop data analytic methods that are consistent with the scientific understanding of the meaning of the $\beta$ coefficients

• Challenges:
  • Time-varying treatment ($A_t, t=1,...,T$)
  • “Independent” variables: $Z_t, S_t, I_t$ that may be affected by prior treatment

• Robustly facilitate noise reduction via use of controls, $Z_t$
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Potential Outcomes

- $\bar{A}_t = \{A_1, A_2, ..., A_t\}$ (random treatments), $\bar{a}_t = \{a_1, a_2, ..., a_t\}$ (realizations of treatments)

- $Y_{t+1}(\bar{a}_t)$ is one potential proximal response

- $I_t(\bar{a}_{t-1})$ is one potential “available for treatment” indicator

- $H_t(\bar{a}_{t-1})$ is one potential history vector
  - $S_t(\bar{a}_{t-1})$ is a vector of features of history $H_t(\bar{a}_{t-1})$
Marginal & Causal Effect

Excursion effect at decision point $t$:

$$E[Y_{t+1}(\bar{A}_{t-1}, 1) - Y_{t+1}(\bar{A}_{t-1}, 0) | I_t(\bar{A}_{t-1}) = 1, S_t(\bar{A}_{t-1})]$$

- Effect is conditional on availability; only concerns the subpopulation of individuals available at decision $t$

- Effect is marginal over any $Y_u, u \leq t, A_u, u < t$ not in $S_t(\bar{A}_{t-1})$---over all variables not in $S_t(\bar{A}_{t-1})$. 

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Marginal & Causal Effect

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Consistency & Micro-Randomized $A_t \rightarrow$

\[
E[Y_{t+1}(\tilde{A}_{t-1}, 1) - Y_{t+1}(\tilde{A}_{t-1}, 0) \mid I_t(\tilde{A}_{t-1}) = 1, S_t(\tilde{A}_{t-1})] = \\
E[E[Y_{t+1} \mid A_t = 1, I_t = 1, H_t] \\
- E[Y_{t+1} \mid A_t = 0, I_t = 1, H_t] \mid I_t = 1, S_t] = \\
E\left[\frac{A_t Y_{t+1}}{p_t(H_t)} - \frac{(1 - A_t) Y_{t+1}}{(1 - p_t(H_t))} \mid I_t = 1, S_t\right]
\]

($p_t(H_t)$ is randomization probability)
Marginal Treatment Effect

Treatment Effect Model:

\[ E[ E[Y_{t+1}|A_t = 1, I_t = 1, H_t] - E[Y_{t+1}|A_t = 0, I_t = 1, H_t]| I_t = 1, S_t] \]

\[ = S_t^T \beta \]

\( H_t \) is participant’s data up to and at time \( t \)
\( S_t \) is a vector of data summaries and time, \( t \), \( (S_t \subseteq H_t) \)
\( I_t \) indicator of availability

We aim to conduct inference about \( \beta \)!
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“Centered and Weighted Least Squares Estimation”

- Simple method for complex data
- Enables unbiased inference for a causal, marginal, treatment effect (the $\beta$’s)
- Inference for treatment effect is not biased by how we use the controls, $Z_t$, to reduce the noise variance in $Y_{t+1}$

https://arxiv.org/abs/1601.00237
Estimation

• Select probabilities: \( \tilde{p}_t(s) \in (0,1) \)

• Form weights: \( W_t = \left( \frac{\tilde{p}_t(S_t)}{p_t(H_t)} \right)^{A_t} \left( \frac{1-\tilde{p}_t(S_t)}{1-p_t(H_t)} \right)^{1-A_t} \)

• Center treatment actions: \( A_t \rightarrow (A_t - \tilde{p}_t(S_t)) \)

• Minimize:

\[
E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta)^2 I_t W_t \right]
\]

• \( E_n \) is empirical distribution over individuals.
Minimize

$$E_n \left[ \sum_{t=1}^{T} \left( Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta \right)^2 I_t W_t \right]$$

Good but incorrect intuition:
Appears to be a weighted “GEE” with a working independence correlation matrix and a centered treatment indicator, $A_t - \tilde{p}_t(S_t)$ thus using the assumption:

- $E[Y_{t+1}|A_t, I_t = 1, Z_t] = Z_t^T \alpha + (A_t - \tilde{p}_t(S_t))S_t^T \beta$

$E_n$ is expectation with respect to empirical distribution
Minimize

\[
E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta)^2 I_t W_t \right]
\]

Good but **incorrect** intuition:

- \( E[Y_{t+1}|A_t, I_t = 1, Z_t] \neq Z_t^T \alpha + (A_t - \tilde{\rho}_t(S_t))S_t^T \beta \)

\( E_n \) is expectation with respect to empirical distribution.
**Minimize**

\[
E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta)^2 I_t W_t \right]
\]

**The Modeling Assumption:**

\[
E[(E[Y_{t+1}|A_t = 1, I_t = 1, H_t] - E[Y_{t+1}|A_t = 0, I_t = 1, H_t])|I_t = 1, S_t] = S_t^T \beta_0
\]

If \( \tilde{\rho}_t \) depends at most on features in \( S_t \), then, under moment conditions, \( \hat{\beta} \) is consistent for \( \beta_0 \).
Theory

Under moment conditions, \( \sqrt{n}(\hat{\beta} - \beta_0) \) converges to a Normal distribution with mean \( \theta \) and var-covar matrix, \((\Sigma_p)^{-1}\Sigma(\Sigma_p)^{-1}\)

\[
\Sigma_p = \mathbb{E}[\sum_{t=1}^{T} \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_tS_tS_t^T]
\]

\(Z_t\) and \(S_t\) are finite dimensional feature vectors.
Gains from Randomization

- Causal inference for a marginal treatment effect
- Inference on treatment effect is robust to working model:

\[ E[Y_{t+1} \mid I_t = 1, H_t] \approx Z_t^T \alpha \]

- \( Z_t \subseteq H_t \)
- Contrast to literature on partially linear, single index models and varying coefficient models
Price due to Marginal Estimand

This “GEE-like” method can only use a working independence correlation matrix

- Estimating function is biased if off-diagonal elements in working correlation matrix: in general,

\[ E\left[ \left( Y_{t+1} - Z_t^T \alpha \right. \right.
\left. - \left( A_t - \tilde{P}_t(S_t) \right) S_t^T \beta \right) I_t W_t I_u W_u \sigma_{t,u} Z_u \right] \neq 0 \]

if \( u \neq t \)
Choice of Weights

Choice of \( \hat{\text{p}}_t(S_t) \) determines marginalization over time under model misspecification of treatment effect.

Example: \( S_t = 1, \hat{\text{p}}_t(S_t) = \bar{\text{p}} \). Resulting \( \hat{\beta} \) is an estimator of

\[
\sum_{t=1}^{T} \frac{E[I_t] \beta_t}{\sum_{t=1}^{T} E[I_t]}
\]

where

\[
\beta_t = E \left[ E[Y_{t+1}|A_t = 1, I_t = 1, H_t] - E[Y_{t+1}|A_t = 0, I_t = 1, H_t]| I_t = 1 \right]
\]
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HeartSteps V1

Heartsteps MRT to Promote Physical Activity Among Sedentary People

Each day of study
Observations are continuous (except self report)
Randomizations to activity prompts occur 5x/day at likely times for increasing physical activity

Next 30 minutes after intervention is delivered
Measured via accelerometer throughout study

Observations
- location (via GPS)
- weather (via internet)
- motion (via wrist band)
- usefulness of prompt (via user indication)
- self report of activity (via app in evenings)

Start Intervention
Prompt planning of next day's activity

Driving?  
Yes → No intervention
No → Walking?
Yes → No intervention
No → Start intervention
Tailored prompt to become physically active

Average 2x/day

Proximal Outcome
physical activity
(steps taken)

Distal Outcome
Overall activity in the 42-day study

PI: P Klasnja
Location: University of Michigan
Funding: NHLBI/NIA R01HL125440
On each of $n=37$ participants:

a) Activity suggestion, $A_t$

- Provide a suggestion with probability .6
  - a tailored sedentary-reducing activity suggestion (probability=.3)
  - a tailored walking activity suggestion (probability=.3)
- Do nothing (probability=.4)

- 5 times per day * 42 days = 210 decision points
Conceptual Models

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t \]
\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \alpha_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

- \( t=1,...,T=210 \)
- \( Y_{t+1} = \) log-transformed step count in the 30 minutes after the \( t^{th} \) decision point,
- \( A_t = 1 \) if an activity suggestion is delivered at the \( t^{th} \) decision point; \( A_t = 0 \), otherwise,
- \( Z_t = \) log-transformed step count in the 30 minutes prior to the \( t^{th} \) decision point,
- \( d_t = \) days in study; takes values in \((0,1,...,41)\)
HeartSteps Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t, \text{ and} \]

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \alpha_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

<table>
<thead>
<tr>
<th>Causal Effect Term</th>
<th>Estimate</th>
<th>95% CI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 A_t )</td>
<td>( \hat{\beta}_0 = .13 )</td>
<td>(-0.01, 0.27)</td>
<td>.06</td>
</tr>
<tr>
<td>(effect of an activity suggestion)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_0 A_t + \beta_1 A_t d_t )</td>
<td>( \hat{\beta}_0 = .51 )</td>
<td>(.20, .81)</td>
<td>&lt;.01</td>
</tr>
<tr>
<td>(time trend in effect of an activity suggestion)</td>
<td>( \hat{\beta}_1 = -0.02 )</td>
<td>(-.03, -.01)</td>
<td>&lt;.01</td>
</tr>
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</table>
On each of $n=37$ participants:

a) Activity suggestion

- Provide a suggestion with probability .6
- a tailored walking activity suggestion (probability=.3)
- a tailored sedentary-reducing activity suggestion (probability=.3)
- Do nothing (probability=.4)

- 5 times per day * 42 days = 210 decision points
HeartSteps Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_{1t} + \beta_1 A_{2t} \]

- \( A_{1t} = 1 \) if walking activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{1t} = 0 \), otherwise,
- \( A_{2t} = 1 \) if sedentary-reducing activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{2t} = 0 \), otherwise,

<table>
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<tr>
<td>( \beta_0 A_{1t} + \beta_1 A_{2t} )</td>
<td>( \hat{\beta}_0 = .21 )</td>
<td>(.04, .39)</td>
<td>.02 ns</td>
</tr>
<tr>
<td></td>
<td>( \hat{\beta}_1 &gt;0 )</td>
<td>ns</td>
<td>ns</td>
</tr>
</tbody>
</table>
Initial Conclusions

• The data indicates that there is a causal effect of the activity suggestion on step count in the succeeding 30 minutes.
  • This effect is primarily due to the walking activity suggestions.
  • This effect deteriorates with time
  • The walking activity suggestion initially increases step count over succeeding 30 minutes by approximately 271 steps but by day 20 this increase is only approximately 65 steps.
Discussion

Problematic Analyses

• GLM & GEE analyses
• Random effects models & analyses
• Machine Learning Generalizations:
  – Partially linear, single index models & analysis
  – Varying coefficient models & analysis

--These analyses do not take advantage of the micro-randomization. Can accidentally eliminate the advantages of randomization for estimating causal effects--
Discussion

• Randomization enhances:
  – Causal inference based on minimal structural assumptions

• Challenge:
  – How to include random effects which reflect scientific understanding ("person-specific" effects) yet not destroy causal inference?
Collaborators!
Minimize

\[ En \left[ \sum_{t=1}^{T} \left( Y_{t+1} - Z_{t}^T \alpha - (A_{t} - \tilde{p}_{t}(S_{t}))S_{t}^T \beta \right)^2 I_{t}W_{t} \right] \]

- Resulting \( \hat{\beta} \) is an estimator of

\[
\left( E \sum_{t=1}^{T} \tilde{p}_{t}(S_{t})(1 - \tilde{p}_{t}(S_{t}))I_{t}S_{t}S_{t}^T \right)^{-1} E \left[ \sum_{t=1}^{T} S_{t}\tilde{p}_{t}(S_{t})(1 - \tilde{p}_{t}(S_{t}))I_{t}\beta_{t}(S_{t}) \right]
\]