50 min. Micro-randomized trials are trials in which individuals are randomized 100's or 1000's of times over the course of the study. The goal of these trials is to assess the impact of momentary interventions, e.g. interventions that are intended to impact behavior over small time intervals. A fast growing area of mHealth concerns the use of mobile devices for both collecting real-time data, for processing this data and for providing momentary interventions. We discuss the design and analysis of these types of trials.
mHealth

- Goal: Design a Continuously Learning Mobile Health Intervention: “HeartSteps”
- “Micro-Randomized” Trial

health domains
Behavior change and maintenance of this change (exercise, healthy eating, sedentary behavior)
Self-management of a chronic disorder (Adherence to meds, adherence to self-care behaviors, mental illness, cognitive support, substance abuse)
Data from wearable devices that sense and provide treatments

\[ S_1, A_1, Y_1, \ldots, S_j, A_j, Y_j, \ldots \]

- \( S_j \): State at \( j^{th} \) decision time (high dimensional)
- \( A_j \): Action at \( j^{th} \) decision time (treatment)
- \( Y_j \): Proximal Response (time-varying response)

Contents of the state are a modeling decision—State is not only what you collect between past decision time and present decision time. It includes summaries of history that are allow you to assume Markovian structure. Can include time of day or day of week and present weather. Volatility in craving over last \( x \) days
Examples

1) Decision Times (Times at which a treatment can be provided.)
   1) Regular intervals in time (e.g. every 10 minutes)
   2) At user demand

HeartSteps includes two sets of decision times
1) Momentary: Approximately every 2-2.5 hours
2) Daily: Each evening at user specified time.

The momentary times were selected because these times are the times at which most people are able to be active
Pre-morning commute, mid-day, mid-afternoon, evening commute, after dinner.

The phone software monitors a risk measure at regular time intervals and if the risk measures hits a criterion then a treatment is provided.
State is not only what you collect between past decision time and present decision time. It includes summaries of history that are allow you to assume Markovian structure. Can include time of day or day of week and present weather.

**Examples**

2) State $S_j$
   1) Passively collected (location, weather, busyness of calendar, social context, activity on device, physical activity)
   2) Actively collected (self-report)

HeartSteps includes activity recognition (walking, driving, standing/sitting), weather, location, calendar, adherence, step count, whether momentary intervention is on, self-report: usefulness, burden, self-efficacy, etc.
Some Treatment types: behavioral, cognitive, motivational, social, self-monitoring, information, alerts

Examples

3) Actions $A_j$
   1) Treatments that can be provided at decision time
   2) Whether to provide a treatment

HeartSteps includes two types of treatments
1) **Momentary** Lock Screen Recommendation
2) **Daily** Activity Planning
Examples

3) Actions $A_j$
   1) Treatments that can be provided at decision time
   2) Whether to provide a treatment

HeartSteps includes two types of treatments
1) **Momentary** Lock Screen Recommendation
2) **Daily** Activity Planning
Daily Activity Planning

How will you be active tomorrow?

No Plan or

- Tomorrow, when I get to work in the morning, I will take a long route to the office to get some extra steps.
- Tomorrow, when I feel like a cup of coffee or tea, I’ll go out to get it at a coffee shop or another building, so I can get a few steps.
- Every time I talk on the phone tomorrow, I will walk around to get some steps and stretch my legs.
- Tomorrow, when I come to work, I’ll park or get off the bus a bit further to get a few extra minutes of walking.

or Generate a plan

Next
The location of the like button biases against the person hitting like.
The snozz button turns off the momentary lock screen recommendations for x hours.

Occurs up to 5 times per day
Examples

4) Proximal Response $Y_j$

HeartSteps: Activity (step count) over next 60 minutes.

Frequently the actions are primarily designed to have a near-term effect on the individual. E.g. Help then manage current craving/stress, help them manage or be aware of the impact of their social setting on their craving/stress.
Types of questions that may be addressed using data from a micro-randomized trial:
1) For a given choice of proximal outcome (e.g., momentary reward) we can eliminate (screen out) ineffective actions. For example we may find that the momentary actions have no impact.
2) Use SNMM to consider moderation by context.

Questions aimed at future trial:
1) Build a predictor of risk for disengagement (need to define what disengagement means). This predictor could function as a warm-start predictor in the online trial.
2) Predictor for when person is available for intervention (?)
3) Build a momentary reward
4) Build a descriptive model using the data and then use simulations based on this descriptive model to compare actor-critic bandit and actor-critic average reward.
Types of questions that may be addressed using data from a micro-randomized trial:
1) For a given choice of proximal outcome (e.g. momentary reward) we can eliminate (screen out) ineffective actions. For example we may find that the momentary actions have no impact.
2) Use SNMM to consider moderation by context.

Questions aimed at future trial:
1) build a predictor of risk for disengagement (need to define what disengagement means). This predictor could function as a warm-start predictor in the online trial.
2) build a warm start policy both with bandit and avg. reward
3) construct a momentary reward
4) build a descriptive model using the data and then use simulations based on this descriptive model to compare actor-critic bandit and actor critic average reward.
People wanting to build a dynamical system model might use “multisine excitation.”

WHAT IF ANYTHING DO WE GAIN BY THINKING OF THIS DESIGN AS A FACTORIAL DESIGN????

Units are people

What are the factors?

What about two time-varying factors (activity planning, lockscreen message). Then the main effects would be time varying. We would have different kinds of interactions as well. Interactions between factors. The new types of effects are delayed effects of one factor on its future effects as well as delayed interactions..... How to handle nested factors?? Note that lockscreen messages on day t are nested within activity planning

So can one think also of average two-way interaction between G and L (activity planning and lockscreen messages). The time scale is different for these two as lockscreen messages can occur 5 times per day whereas activity planning can only appear once per day.
Types of questions that may be addressed using data from a micro-randomized trial:

1) For a given choice of proximal outcome (e.g., momentary reward) we can eliminate (screen out) ineffective actions. For example, we may find that the momentary options have no impact.

2) Use SNMM to consider moderation by context.

Questions aimed at future trial:

1) Build a predictor of risk for disengagement (need to define what disengagement means). This predictor could function as a warm-start predictor in the online trial.

2) Build a warm start policy both with bandit and avg. reward.

3) Construct a momentary reward.

4) Build a descriptive model using the data and then use simulations based on this descriptive model to compare actor-critic bandit and actor critic average reward.
planned study 2160 decision times.
J=42*5=210 momentary randomizations
Available

- $A_j$ is only delivered if the intervention is on at decision time $j$.

- Set $I_j = 1$ if the intervention is on at decision time $j$, otherwise $I_j = 0$

The momentary intervention can be turned off for 4 or 8 hours by the participant. The intervention is also off if the participant is classified as currently active (e.g., walking) or classified as currently driving.
Potential Outcomes

- Define
  \[ \tilde{A}_j = \{A_1, A_2, \ldots, A_j\}, \tilde{a}_j = \{a_1, a_2, \ldots, a_j\} \]

- Define \( Y_j(\tilde{a}_j) \) to be the observed response, \( Y_j \) if \( \tilde{A}_j = \tilde{a}_j \), e.g., \( Y_j = Y_j(\tilde{A}_j) \)

- Define \( I_j(\tilde{a}_{j-1}) \) to be the observed “intervention on” indicator if \( \tilde{A}_{j-1} = \tilde{a}_{j-1} \)
Marginal over randomization treatment policy (and effects thereof), conditional on those who have intervention on.

The group who have the intervention turned on is a selected group of people likely depending on the intervention dose they experienced up to time j. This intervention dose \( \bar{A}_{j-1} \) may have caused burden, may have caused learning.

**Proximal Main Effect**

- Define the Proximal Main Effect at time \( j \) as
  
  \[
  E[Y_j(\bar{A}_{j-1}, 1) - Y_j(\bar{A}_{j-1}, 0)|I_j(\bar{A}_{j-1}) = 1]
  \]

- What does this estimand mean?
**Proximal Main Effect**

- The randomization implies that
  \[
  E[Y_j(\bar{A}_{j-1}, 1) - Y_j(\bar{A}_{j-1}, 0)|I_j(\bar{A}_{j-1}) = 1] = \\
  E[Y_j|I_j = 1, A_j = 1] - E[Y_j|I_j = 1, A_j = 0]
  \]

- Put
  \[
  \beta(j) = E[Y_j|I_j = 1, A_j = 1] - E[Y_j|I_j = 1, A_j = 0]
  \]

Assuming “consistency” THESE ARE CAUSAL EFFECTS!!!!!
Proposal

Design and size micro-randomized trial to detect proximal main effect of treatment

The proximal main effect is a time-varying main effect $\beta(j), j=1, \ldots, J$

The proximal main effect is a causal effect.

Main effects are marginal effects!

Delayed effects which are akin to higher order interactions would be investigated in secondary analyses
Primary Analysis Tests for Parametric Alternative

- We construct a test statistic for testing $H_0 : \beta(j) = 0, \forall j$

- HeartSteps parametric alternative:
  $$\beta(j) = \beta_0 + \beta_1 \left\lfloor \frac{j-1}{5} \right\rfloor + \beta_2 \left\lfloor \frac{j-1}{5} \right\rfloor^2$$

  $\rightarrow$ test
  $$H_0 : \beta_i = 0, i = 0, 1, 2$$

Since the model for the proximal effect of $A_j$ on $Y_j$ does not depend on time of day, we are averaging any variation in proximal effect across the occasions during the day (recall we are sizing the study; a primary analysis might be a little more complex and in secondary data analyses one would likely estimate and test if the proximal effect varies by time of day and/or varies by $j$, since $j$ denotes duration in study).
Test Statistic for Sample Size Calculation

- Test statistic based on fitted model:

\[ E[Y_j | I_j = 1, A_j] = \gamma(j) + \beta(j)(A_j - q_j) \]

where \( q_j \) is the randomization probability
- \( q_j = .4 \) in HeartSteps

Centering of \( A_j \) protects estimation of \( \beta(j) \) from misspecification of \( \gamma(j) \). --This works for consistency of \( \hat{\beta}(j) \) and variance estimation in the data analysis but when we do the sample size formula we simplify the variance—the simplification assumes \( \gamma \) is correctly specified.
Test Statistic for Sample Size Calculation

- Test statistic is based on least squares fit of
  \[
  E[Y_j | I_j = 1, A_j] = \gamma(j) + \beta(j)(A_j - q_j)
  \]
  HeartSteps:
  \[
  \beta(j) = \beta_0 + \beta_1 \left[ \frac{j-1}{5} \right] + \beta_2 \left[ \frac{j-1}{5} \right]^2
  \]
- You select parameterization of \(\gamma(j)\)

least squares acting as if time points within a person are independent.
Alternative for Sample Size Calculation

- One calculates a sample size to detect a given alternative with a given power.

- Alternative:
  \[ H_1 : \beta_i = d_i \sigma, i = 0, 1, 2 \]
  where \( \sigma^2 \) is the conditional variance.
Alternative for Sample Size Calculation

• Conditional variance is

\[ \sigma^2 = \text{VAR}(Y_j | I_j = 1, A_j) \]
Specify alternative so that scientist can provide $d_i$’s
$J=5*7*6$ in our study
Test Statistic for Sample Size Calculation

- Put \( Y_i = (Y_{i1}, \ldots, Y_{iJ})^T \) for \( i^{th} \) subject

\( p \) is the total number of parameters \((p > 3)\);
\( X_i \) is the associated design matrix \((J \text{ by } p)\)
\( N \) is sample size

Last 3 columns of \( X_i \) contain row entries:
\( I_{ij}(A_{ij} - q_{ij}), I_{ij}(A_{ij} - q_{ij})\left[\frac{j-1}{5}\right], \)
\( I_{ij}(A_{ij} - q_{ij})\left[\frac{j-1}{5}\right]^2 \)

least squares acting as if time points within a person are independent.
Test Statistic for Sample Size Calculation

- “GEE” test statistic is

\[ N\hat{\beta}^T (K\hat{\Sigma}K^T)^{-1}\hat{\beta} \]

where \( \hat{\Sigma} \) is the usual sandwich estimator of the variance-covariance and \( K \) is a \( 3 \) by \( p \) matrix picking out columns associated with coefficients \( \beta \).
Sample Size Calculation

Under standard moment assumptions, the asymptotic distribution is a Chi-Squared on 3 degrees of freedom with non-centrality parameter: $d^T (\Sigma_\beta)^{-1} d$

$\Sigma_\beta = (Z^T Z)^{1} \text{divided by } q(1-q)\tau$ $Z$ is number of time points by 3. The 3 columns correspond to the columns $[A_j-.4, (A_j-.4) \lfloor \frac{j-1}{5} \rfloor, (A_j-.4) \lfloor \frac{j-1}{5} \rfloor^2]$ in the design matrix.
Working Assumptions for Sample Size Calculation

1) The model errors, \((\epsilon_{ij}, \epsilon_{ik})\) are uncorrelated with the treatments \((A_{ij}, A_{ik})\) given \((I_{ij} = 1, I_{ik} = 1)\)

2) \(E(\epsilon_{ij} \mid I_t = 1, A_t) = 0\)

3) \(Var(\epsilon_{ij} \mid I_t = 1, A_t)\) is constant.

\[
\epsilon_{ij} = Y_{ij} - \left( \gamma(j) + \beta(j)(A_{ij} - q_{ij}) \right)
\]
Sample Size Calculation

- Under the working assumptions, $\Sigma_\beta$ only depends on polynomials in $[\frac{j-1}{5}]$, the marginal distribution of $I_j$ and on the randomization probabilities.

- The asymptotic distribution of the test statistic does not depend on the form of $\gamma(j)$

\[ \Sigma_\beta = (Z^T Z)^{\frac{1}{2}} \text{ divided by } q(1-q)\tau \quad Z \text{ is number of time points by 3. The 3 columns correspond} \]

to the columns $[A_{j-.4}, (A_{j-.4}) \lfloor\frac{j-1}{5}\rfloor, (A_{j-.4}) \lfloor\frac{j-1}{5}\rfloor^2]$ in the design matrix.
The contrasts become within person contrasts due to the assumption of smoothness over time. If the proximal effect at each time were to be estimated separately then it would be like a two arm study at each time j.

$$\Sigma_{\beta} = (Z^T Z)^{-1} \text{ divided by } q(1-q)\tau$$

$Z$ is number of time points by 3. The 3 columns correspond to the columns $[A_j-0.4, (A_j-0.4) \lfloor \frac{j-1}{5} \rfloor, (A_j-0.4) \lfloor \frac{j-1}{5} \rfloor^2]$ in the design matrix.
Sample Size Calculation

- Instead of a Chi-Squared on 3 degrees we use \( \frac{3(N-p+2)}{N-p} F_{p,N-p} \) with the same noncentrality parameter \( d^T (\Sigma_\beta)^{-1} d \)

\( p \) is the number of parameters in linear model. These are the degrees of freedom recommended by Lloyd A. Mancl and Timothy A. DeRouen A Covariance Estimator for GEE with Improved Small-Sample Properties BIOMETRICs 57 , 126-134 March 2001

\( \Sigma_\beta = (Z^T Z)^{-1} \) divided by \( q(1-q)\tau \) where \( Z \) is number of time points by 3. The 3 columns correspond to the columns \( \frac{A_j-.4}{5}, \frac{(A_j-.4)\lfloor j-1/5 \rfloor}{5}, \frac{(A_j-.4)\lfloor j-1/5 \rfloor^2}{5} \) in the design matrix.
HeartSteps Example

- Standardized $d_i$’s
  - initial proximal effect: $d_0 = 0$
  - output average proximal effect
  - day of maximal proximal effect: $\frac{-d_1}{2d_2} = 28$

- Model:
  $$\gamma(j) + \beta(j)(A_{ij} - .4), \ j = 1, \ldots, 42$$

where
  $$\gamma(j) = \gamma_0 + \gamma_1 \lfloor \frac{j-1}{5} \rfloor + \gamma_2 \left( \frac{j-1}{5} \right)^2$$

Specify alternative so that scientist can provide $d_i$’s

42 days in our study
Average proximal effect is standardized.
#parameters=6

Meaningful increase in stepcount is 1000/day
Usual std is 2000/day
Roughly a standardized treatment effect of 200/666= .3
Primary Data Analysis

• Put \( Y_i = (Y_{i1}, \ldots, Y_{iJ})^T \) for \( i^{th} \) subject

\( p \) is the total number of parameters \((p > 3)\);
\( X_i \) is the associated design matrix \((J \text{ by } p)\)
\( N \) is sample size

Last 3 columns of \( X_i \) contain row entries:
\[ I_{ij}(A_{ij} - q_{ij}), I_{ij}(A_{ij} - q_{ij})\left[\frac{j-1}{5}\right], I_{ij}(A_{ij} - q_{ij})\left[\frac{j-1}{5}\right]^2 \]

GEE here is least squares acting as if time points within a person are independent.
Test Statistic

- “GEE” test statistic is

\[ N \hat{\beta}^T (K \hat{\Sigma} K^T)^{-1} \hat{\beta} \]

where \( K \) is a 3 by \( p \) matrix picking out columns associated with \( \beta \) coefficients
GEE here is least squares acting as if time points within a person are independent. Lloyd A. Mancl and Timothy A. DeRouen A Covariance Estimator for GEE with Improved Small-Sample Properties BIOMETRICs 57 , 126-134 March 2001

The resulting estimator of the variance of $\hat{\beta}$ is consistent. We are not assuming that $\gamma(j)$ is correct.
Simulation Results
Type 2 Error Rate (2000 data sets)

<table>
<thead>
<tr>
<th>Average Proximal Effect (Sample Size)</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05(115)</td>
<td>0.790</td>
</tr>
<tr>
<td>0.06(81)</td>
<td>0.794</td>
</tr>
<tr>
<td>0.07(61)</td>
<td>0.800</td>
</tr>
<tr>
<td>0.08(48)</td>
<td>0.801</td>
</tr>
<tr>
<td>0.09(39)</td>
<td>0.798</td>
</tr>
<tr>
<td>0.10(33)</td>
<td>0.803</td>
</tr>
</tbody>
</table>

E[I]=0.7, no initial effect, maximal effect on day 29, randomization prob.=0.4

Simulations indicate:
Method is sensitive to
Guess of average amount of time intervention is on: $\frac{1}{J} \sum_{j=1}^{J} E[I_j]$. Choose on the low side to be safe.

Guess of average proximal txt effect. Choose on the low side to be safe.

Heteroscedasticity of errors variance of Y when A=1 is larger than variance of Y when A=0 is problematic

Simulations indicate robustness to
$I_{j+1}$ a function of past $A_j$'s

Guess at day of maximal proximal effect (we use different function from quadratic when this day is less than ½ of the way through the study—this is not presented here)

Non-symmetry or skewness to residual error distribution.
Positive correlated across time residual errors
Mixture of people, some of whom have the intervention turned off x % of time and some who have their intervention turned off y% of the time where overall % time turned off is .7 or .5
Scientific Goals

1) Develop methods/trial designs for assessing if there are proximal causal effects of the actions on the response.

2) Develop methods for assessing if there are delayed causal effects; assess if the proximal or delayed causal effects vary by particular state variables.

3) Develop data methods for constructing a treatment policy that inputs state and delivers actions via phone.

4) Develop online training algorithms that will result in a “Continually Updating” treatment policy.
Collaborators: P. Liao, A. Lee, C. Anderson, P. Klasnja & A. Tewari

Sample size calculator at
https://jisun.sinyapps.io/SampleSizeCalculator/

Email if you have questions!

samurphy@umich.edu