Time-Varying Causal Treatment Effects

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Outline

- Introduction to mobile health
- Causal Treatment Effects (aka Causal Excursions)
- (A wonderfully simple) Estimation Method
- HeartSteps
HeartSteps

Context provided via data from:

- **Wearable band** → activity and sleep quality;
- **Smartphone sensors** → busyness of calendar, location, weather;
- **Self-report** → stress, user burden

In which contexts should the smartphone provide the user with a tailored activity suggestion?
Data from wearable devices that sense and provide treatments

- On each individual: $O_1, A_1, Y_2, \ldots, O_t, A_t, Y_{t+1}, \ldots$

- $t$: Decision point
- $O_t$: Observations at $t^{th}$ decision point (high dimensional)
- $A_t$: Action at $t^{th}$ decision point (treatment)
- $Y_{t+1}$: Proximal response (e.g., reward, utility, cost)
Structure of Mobile Health Intervention

1) Decision Points: times, $t$, at which a treatment might be provided.
   1) Regular intervals in time (e.g. every minute)
   2) At user demand

Heart Steps: approximately every 2-2.5 hours: pre-morning commute, mid-day, mid-afternoon, evening commute, after dinner
Structure of Mobile Health Intervention

2) Observations $O_t$
   1) Passively collected (via sensors)
   2) Actively collected (via self-report)

Heart Steps: classifications of activity, location, weather, step count, busyness of calendar, user burden, adherence…….
3) Actions $A_t$
   1) Types of treatments/engagement strategies that can be provided at a decision point, $t$
   2) Whether to provide a treatment

*HeartSteps*: tailored activity suggestion (yes/no)
Availability

- Treatments, $A_t$, can only be delivered at a decision point if an individual is available. 
  - $O_t$ includes $I_t=1$ if available, $I_t=0$ if not

- Treatment effects at a decision point are conditional on availability.

- Availability is not the same as adherence!
Structure of Mobile Health Intervention

4) Proximal Outcome $Y_{t+1}$

Heart Steps: Step count over 30 minutes following decision point, $t$
Continually Learning Mobile Health Intervention

1) Trial Designs: Are there effects of the actions on the proximal response? *experimental design*

2) Data Analytics for use with trial data: Do effects vary by the user’s internal/external context? Are there delayed effects of the actions? *causal inference*

3) Learning Algorithms for use with trial data: Construct a “warm-start” treatment policy. *batch Reinforcement Learning*

4) Online Algorithms that personalize and continually update the mHealth Intervention. *online Reinforcement Learning*
Micro-Randomized Trial Data

On each of \( n \) participants and at each of \( t=1,...,T \) decision points:

- \( O_t \) observations at decision point \( t \),
  - includes \( I_t=1 \) if available, \( I_t=0 \) if not
- \( A_t=1 \) if treated, \( A_t=0 \) if not treated at decision \( t \)
  - Randomized, \( p_t(H_t) = P[A_t=1| H_t, I_t=1] \)
- \( Y_{t+1} \) proximal response

\[ H_t = \{(O_i, A_i, Y_{i+1}), i=1,...,t-1; O_t\} \] denotes data through \( t \)
Conceptual Models

Generally data analysts fit a series of increasingly more complex models:

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

and then next,

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

and so on...

- \( Y_{t+1} \) is activity over 30 min. following \( t \)
- \( A_t = 1 \) if activity suggestion and 0 otherwise
- \( Z_t \) summaries formed from \( t \) and past/present observations
- \( S_t \) potential moderator (e.g., current weather is good or not)
Conceptual Models

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\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

and so on...

\[ \alpha_1^T Z_t \] is used to reduce the noise variance in \( Y_{t+1} \)

\( Z_t \) is sometimes called a vector of control variables
Causal, Marginal Effects

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t \]

\( \beta_0 \) is the effect, marginal over all observed and all unobserved variables, of the activity suggestion on subsequent activity.

\[ Y_{t+1} \sim \alpha_0 + \alpha_1^T Z_t + \beta_0 A_t + \beta_1 A_t S_t \]

\( \beta_0 + \beta_1 \) is the effect when the weather is good \((S_t=1)\), marginal over other observed and all unobserved variables, of the activity suggestion on subsequent activity.
Goal

• Develop data analytic methods that are consistent with the scientific understanding of the meaning of the $\beta$ coefficients

• Challenges:
  • Time-varying treatment ($A_t$, $t=1,\ldots,T$)
  • “Independent” variables: $Z_t$, $S_t$, $I_t$ that may be affected by prior treatment

• Robustly facilitate noise reduction via use of controls, $Z_t$
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Potential Outcomes

• $\bar{A}_t = \{A_1, A_2, \ldots, A_t\}$ (random treatments), $\bar{a}_t = \{a_1, a_2, \ldots, a_t\}$ (realizations of treatments)

• $Y_{t+1}(\bar{a}_t)$ is one potential proximal response

• $I_t(\bar{a}_{t-1})$ is one potential “available for treatment” indicator

• $H_t(\bar{a}_{t-1})$ is one potential history vector
  - $S_t(\bar{a}_{t-1})$ is a vector of features of history $H_t(\bar{a}_{t-1})$
Excursion effect at decision point $t$:

$$E [Y_{t+1}(\tilde{A}_{t-1}, 1) - Y_{t+1}(\tilde{A}_{t-1}, 0) \mid I_t(\tilde{A}_{t-1}) = 1, S_t(\tilde{A}_{t-1})]$$

- Effect is marginal over any $Y_u, u \leq t, A_u, u < t$ not in $S_t(\tilde{A}_{t-1})$---over all variables not in $S_t(\tilde{A}_{t-1})$.

- Effect is conditional on availability; only concerns the subpopulation of individuals available at decision $t$
Consistency & Micro-Randomized $A_t \rightarrow$

$$E[Y_{t+1}(\bar{A}_{t-1},1) - Y_{t+1}(\bar{A}_{t-1},0) \mid I_t(\bar{A}_{t-1}) = 1, S_t(\bar{A}_{t-1})]$$

$$= E\left[ E[Y_{t+1} \mid A_t = 1, I_t = 1, H_t] \right.$$ 
$$- E[Y_{t+1} \mid A_t = 0, I_t = 1, H_t]| I_t = 1, S_t]$$

$$= E\left[ \frac{A_t Y_{t+1}}{p_t(H_t)} - \frac{(1 - A_t) Y_{t+1}}{(1 - p_t(H_t))} \mid I_t = 1, S_t \right]$$

$(p_t(H_t)$ is randomization probability)$
Marginal Treatment Effect

Treatment Effect Model:

\[
E\left[ E\left[ Y_{t+1} | A_t = 1, I_t = 1, H_t \right] \right]
- E\left[ Y_{t+1} | A_t = 0, I_t = 1, H_t \right]| I_t = 1, S_t ]
= S_t^T \beta
\]

\(H_t\) is participant’s data up to and at time \(t\)

\(S_t\) is a vector of data summaries and time, \(t\), \((S_t \subseteq H_t)\)

\(I_t\) indicator of availability

We aim to conduct inference about \(\beta\)!
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“Centered and Weighted Least Squares Estimation”

• Simple method for complex data
• Enables unbiased inference for a causal, marginal, treatment effect (the $\beta$’s)
• Inference for treatment effect is not biased by how we use the controls, $Z_t$, to reduce the noise variance in $Y_{t+1}$

https://arxiv.org/abs/1601.00237
Estimation

- Select probabilities: $\tilde{p}_t(s) \in (0,1)$
- Form weights: $W_t = \left(\frac{\tilde{p}_t(S_t)}{p_t(H_t)}\right)^{A_t} \left(\frac{1-\tilde{p}_t(S_t)}{1-p_t(H_t)}\right)^{1-A_t}$
- Center treatment actions: $A_t \rightarrow (A_t - \tilde{p}_t(S_t))$

Minimize:

$$E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta)^2 I_t W_t \right]$$

- $E_n$ is empirical distribution over individuals.
Minimize

\[ E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta)^2 I_t W_t \right] \]

Good but incorrect intuition:

- Weighted “GEE” with a working independence correlation matrix and a centered treatment indicator, \( A_t - \tilde{p}_t(S_t) \)
- \( E[Y_{t+1} | A_t, I_t = 1, Z_t] \neq Z_t^T \alpha + (A_t - \tilde{p}_t(S_t))S_t^T \beta \)

\( E_n \) is expectation with respect to empirical distribution.
Minimize

\[ E_n \left[ \sum_{t=1}^{T} (Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta)^2 I_t W_t \right] \]

The Modeling Assumption:

\[
E \left[ (E[Y_{t+1}|A_t = 1, I_t = 1, H_t] \\
- E[Y_{t+1}|A_t = 0, I_t = 1, H_t])|I_t = 1, S_t \right] = S_t^T \beta_0
\]

If \( \tilde{\rho}_t \) depends at most on features in \( S_t \), then, under moment conditions, \( \hat{\beta} \) is consistent for \( \beta_0 \)
Theory

Under moment conditions, \( \sqrt{n}(\hat{\beta} - \beta_0) \) converges to a Normal distribution with mean \( 0 \) and var-covar matrix, \((\Sigma_p)^{-1} \Sigma (\Sigma_p)^{-1}\)

\[
\Sigma_p = E[\sum_{t=1}^{T} \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_tS_tS_t^T]
\]

\(Z_t\) and \(S_t\) are finite dimensional feature vectors.
Gains from Randomization

• Causal inference for a marginal treatment effect
• Inference on treatment effect is robust to working model:

\[ E[Y_{t+1} \mid I_t = 1, H_t] \approx Z_t^T \alpha \]

- \( Z_t \subseteq H_t \)
- Contrast to literature on partially linear, single index models and varying coefficient models
Price due to Marginal Estimand

This “GEE-like” method can only use a working independence correlation matrix

– Estimating function is biased if off-diagonal elements in working correlation matrix: in general,

\[
E\left[ \left( Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{\rho}_t(S_t))S_t^T \beta \right) I_t W_t I_u W_u \sigma_{t,u} Z_u \right] \neq 0
\]

if \( u \neq t \)
Choice of Weights

Choice of \( \tilde{\rho}_t(S_t) \) determines marginalization over time under model misspecification of treatment effect.

Example: \( S_t = 1, \tilde{\rho}_t(S_t) = \tilde{\rho} \). Resulting \( \hat{\beta} \) is an estimator of

\[
\sum_{t=1}^{T} E[I_t] \beta_t / \sum_{t=1}^{T} E[I_t]
\]

where

\[
\beta_t = E[ E[Y_{t+1}|A_t = 1, I_t = 1, H_t] - E[Y_{t+1}|A_t = 0, I_t = 1, H_t]| I_t = 1 ]
\]
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Heart Steps Pilot Study

On each of $n=37$ participants:

a) Activity suggestion, $A_t$
   - Provide a suggestion with probability .6
     - a tailored sedentary-reducing activity suggestion (probability=.3)
     - a tailored walking activity suggestion (probability=.3)
   - Do nothing (probability=.4)

- 5 times per day $\times$ 42 days = 210 decision points
Conceptual Models

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t \]

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \alpha_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

- \( t=1, \ldots T=210 \)
- \( Y_{t+1} \) = log-transformed step count in the 30 minutes after the \( t \)th decision point,
- \( A_t = 1 \) if an activity suggestion is delivered at the \( t \)th decision point; \( A_t = 0 \), otherwise,
- \( Z_t \) = log-transformed step count in the 30 minutes prior to the \( t \)th decision point,
- \( d_t \) = days in study; takes values in \((0,1,\ldots,41)\)
Pilot Study Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_t, \text{ and} \]

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \alpha_2 d_t + \beta_0 A_t + \beta_1 A_t d_t \]

<table>
<thead>
<tr>
<th>Causal Effect Term</th>
<th>Estimate</th>
<th>95% CI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 A_t )</td>
<td>( \hat{\beta}_0 = .13 )</td>
<td>(-0.01, 0.27)</td>
<td>.06</td>
</tr>
<tr>
<td>(effect of an activity suggestion)</td>
<td></td>
<td></td>
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<tr>
<td>( \beta_0 A_t + \beta_1 A_t d_t )</td>
<td>( \hat{\beta}_0 = .51 )</td>
<td>(.20, .81)</td>
<td>&lt;.01</td>
</tr>
<tr>
<td>(time trend in effect of an activity suggestion)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \hat{\beta}_1 = -.02 )</td>
<td>(-.03, -.01)</td>
<td>&lt;.01</td>
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Heart Steps Pilot Study

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     - Do nothing (probability=.4)

• 5 times per day * 42 days = 210 decision points
Pilot Study Analysis

\[ Y_{t+1} \sim \alpha_0 + \alpha_1 Z_t + \beta_0 A_{1t} + \beta_1 A_{2t} \]

- \( A_{1t} = 1 \) if walking activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{1t} = 0 \), otherwise,
- \( A_{2t} = 1 \) if sedentary-reducing activity suggestion is delivered at the \( t^{th} \) decision point; \( A_{2t} = 0 \), otherwise,

<table>
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<td>( \beta_0 A_{1t} + \beta_1 A_{2t} )</td>
<td>( \hat{\beta}_0 = .21 )</td>
<td>(.04, .39)</td>
<td>.02</td>
</tr>
<tr>
<td></td>
<td>( \hat{\beta}_1 &gt; 0 )</td>
<td>ns</td>
<td>ns</td>
</tr>
</tbody>
</table>


Initial Conclusions

• The data indicates that there is a causal effect of the activity suggestion on step count in the succeeding 30 minutes.
  • This effect is primarily due to the walking activity suggestions.
  • This effect deteriorates with time.
  • The walking activity suggestion initially increases step count over succeeding 30 minutes by \( \approx 271 \) steps but by day 21 this increase is only \( \approx 65 \) steps.
Discussion

Problematic Analyses

• GLM & GEE analyses
• Random effects models & analyses
• Machine Learning Generalizations:
  – Partially linear, single index models & analysis
  – Varying coefficient models & analysis

--These analyses do not take advantage of the micro-randomization. Can accidentally eliminate the advantages of randomization for estimating causal effects--
Discussion

• Randomization enhances:
  – Causal inference based on minimal structural assumptions

• Challenge:
  – How to include random effects which reflect scientific understanding (“person-specific” effects) yet not destroy causal inference?
Collaborators!
mHealth

Sense2Stop Smoking Cessation Coach

- Wearable wrist/chest bands provide multiple physiological sensor streams... craving, burden,....

- Supportive stress-regulation interventions available on smartphone 24/7

- In which contexts should the wrist band provide reminder to access stress-regulation apps?
Minimize

\[ E_n \left[ \sum_{t=1}^{T} \left( Y_{t+1} - Z_t^T \alpha - (A_t - \tilde{p}_t(S_t))S_t^T \beta \right)^2 I_t W_t \right] \]

- Resulting \( \hat{\beta} \) is an estimator of

\[ \left( E \sum_{t=1}^{T} \tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_tS_tS_t^T \right)^{-1} E \left[ \sum_{t=1}^{T} S_t\tilde{p}_t(S_t)(1 - \tilde{p}_t(S_t))I_t\beta_t(S_t) \right] \]