Math 104: Homework 3 (due February 10)

1. Ross exercise 8.4
2. Ross exercise 8.5
3. Ross exercise 9.6

4. Let \((a_n)\) be a sequence defined according to \(a_1 = t\) where \(t > 0\), and \(a_{n+1} = 2a_n/(1 + a_n)\) for \(n \in \mathbb{N}\). Prove that \(a_n \to 1\) as \(n \to \infty\).

5. Let \((s_n)\) and \((t_n)\) be Cauchy sequences defined on \(\mathbb{R}\), and let \((u_n)\) be a sequence defined as \(u_n = as_n + bt_n\) for all \(n\), where \(a, b \in \mathbb{R}\). By using the definition of a Cauchy sequence only, without assuming that limits of \((s_n)\) and \((t_n)\) exist, prove that \((u_n)\) is a Cauchy sequence.

6. Ross exercise 9.10
7. Ross exercise 10.8

8. Let \((s_n)\) be a sequence defined for \(n \in \mathbb{N}\) as

\[
 s_n = \begin{cases} 
 1 + \frac{1}{n} & \text{for } n \text{ odd,} \\
 -1 & \text{for } n \text{ even.}
\end{cases}
\]

Calculate the monotonic sequences

\[
 u_N = \inf\{s_n : n > N\}, \quad v_N = \sup\{s_n : n > N\}
\]

for each \(N \in \mathbb{N}\) and hence determine \(\lim \inf s_n\) and \(\lim \sup s_n\).