Real Semantics: Capturing Floating-Point Imprecision
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BACKGROUND

• Floating-point numbers are ubiquitous in computing applications
• Programmers usually treated them just as real numbers but they are not real!

Fig. 1. IEEE 754 single-precision floating-point format (Wikipedia [3]).

• Due to the finite, discrete binary construction of floating-point numbers, simple real numbers like 0.1 in decimal cannot be represented in exact form in floating-point
• Floating-point numbers are more “dense” around zero and are relatively sparse at higher orders of magnitude
• Arithmetic operations over floating-point numbers are not closed, which means an arithmetic operation involving two valid floating-point numbers may end up with a result that doesn’t have an exact floating-point representation and therefore rounding is required
• Rounding and cancellation are sources of potentially disastrous imprecision

MOTIVATION

There are existing techniques to analyze or optimize floating-point usage in computer programs, with some limitations:
• Formal verification: Good at catching erroneous conditions like divide-by-zero exceptions
• Not sophisticated enough to deal with imprecisions
• Semantics of floating-point arithmetic are too difficult to model and check formally
• Static analysis tools (e.g. Herbie [2])
• Can optimize floating-point usage in mathematical expressions
• Not general enough to enable generic full-program analyses

We decide to approach this problem by dynamic analysis!

IMPLEMENTATION

Real Semantics: a custom LLVM IR interpreter engine that augments floating-point operations with arbitrary precision arithmetic (provided by GNU MPFR).

We choose LLVM IR for generality: any existing program source code that the LLVM front-end can handle can be compiled down to IR.

We introduce a new data type, namely SmartFloat, in LLVM IR, which holds two representations for each floating-point value in the program:
• regular precision – native float or double type
• high precision – mpfr-real type
SmartFloat is bigger than native floating-point numbers, so we store them in a separate map, keyed by their address in the program’s native address space.

We check for precision loss by comparing the two representations in the SmartFloat object. To minimize false positives/extra error messages, we only report an error when:
• imprecision results in divergence in control
• imprecision results in divergence in external effects (e.g. output, uninterpreted library calls, etc.)

```c
// more operator definitions...
SmartFloat::real<PRECISION>
SmartFloat::real<PRECISION>::smartfloat::real
SmartFloat::real<PRECISION>::smartfloat::real::operator+=
int SmartFloat::operator+(SmartFloat::real<PRECISION>&...
SmartFloat::real<PRECISION>::smartfloat::real::operator-
```

Fig. 2. Definition of the new SmartFloat LLVM IR data type.

RESULTS

just problemd/float pa, float pb; {
float f1 = pa + pb - pb + pa; std::printf("\%.8f\n", f1);
float f2 = (1.0 - pa) * (1.0 - pb); std::printf("\%.8f\n", f2);
return 0;
}

float f = pa + pb - pb + pa;
float result = 0.0;
float step = 0.000001;
for (x = lower; x < upper; x += step) {
result = result + step;
printf("\%0.8f\n", result);
}

```c
#include math.h
#include stdio.h
```

```c
float upper = 1.0; float lower = 0.0;
float step = 0.0001;
float result = 0.0;
float f1 = pa + pb - pb + pa;
```

Fig. 3. High-level system architecture.

```c
just problemd/float pa, float pb; {
float f1 = pa + pb - pb + pa; std::printf("\%.8f\n", f1);
float f2 = (1.0 - pa) * (1.0 - pb); std::printf("\%.8f\n", f2);
return 0;
}
```

Fig. 4. A sample program and its output when running with our tool when we supply `pa = 1e-6` and `pb = 2e-2` as the input. No error messages show up before the final output, which means the f1 satisfies the precision requirement.

CONCLUSIONS

• Real Semantics is a custom LLVM IR interpreter engine that can dynamically find floating-point imprecision errors in computer programs.
• It augments regular floating-point operations with arbitrary precision arithmetic, and it checks for precision loss by comparing the two representations of the same floating-point value.
• For better usability, we only report errors when imprecision results in divergent program control flow or external effects.

Future Work
• Improve performance by using just-in-time compilation or instrumentation
• The interpreter alone incurs a 200-1000x slowdown
• Better precision measurement by using “real” arbitrary precision arithmetic libraries (like those used by computer algebraic systems) in place of MPFR

REFERENCES


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Check out source code at https://github.com/darkin/real-semantics